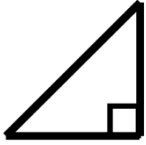**Module 5 The Pythagorean Theorem**

Section 5.1 Acute, Obtuse, and Right Triangles
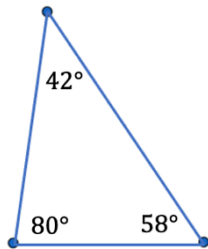Practice Problems 5.1
For Problem 1-4, tell whether the triangle given is right, acute, or obtuse.
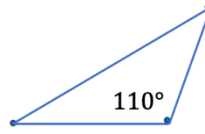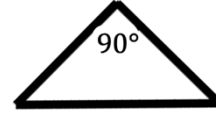
1.   2.   3.   4.
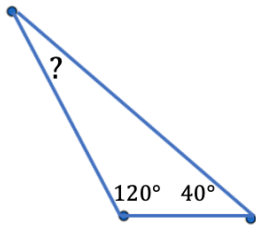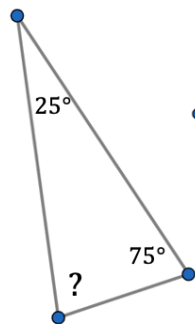
90°

42°

110°

80°   58°

For Problem 5-8, find the measure of the missing angle in the triangle given.

5.   6.   7.   8.

?   115°   ?

25°   35°   ?   90°   23°

120° 40°   75°
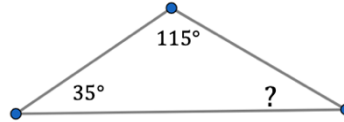
?
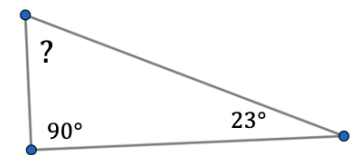
For Problem 9-12, find the third angle in each triangle given the other two angles and tell if the triangle is right, acute, or obtuse.

9.   33° and 22° and _____°   10.   40° and 40° and _____°

_____ triangle   _____ triangle

11.   57° and 62° and _____°   12.   52° and 38° and _____°

_____ triangle   _____ triangle

13.   Two angles of a triangle are 47° and 24°. Shonda said the triangle is obtuse. How did she know?

14.   Why can there only be one obtuse angle in a triangle?

Section 5.2 Right Triangles and the Pythagorean Theorem
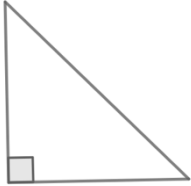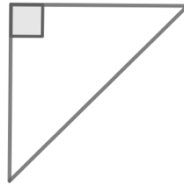Practice Problems 5.2

For Problem 1-6, identify the hypotenuse and legs of the triangle. Mark the hypotenuse with an h and each leg with an l.
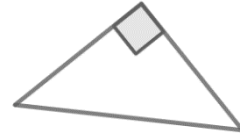
1.

2.

3.

4.

5.
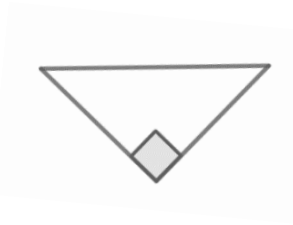
6.
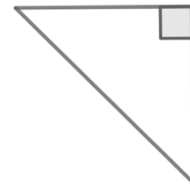
For Problem 7-10, tell whether or not the triangle given is a right triangle. Explain why or why not.

7.

17
8
15

8.

25
7
24

9.

8
4
7
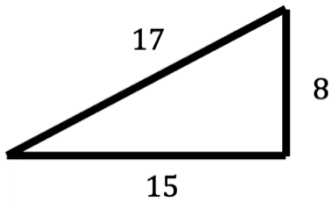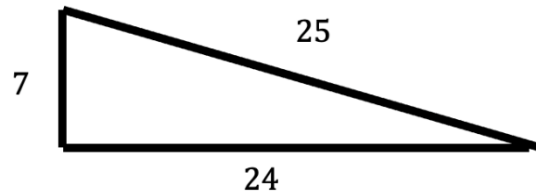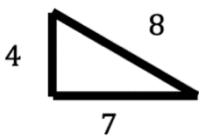
10.

For Problem 11-15, solve the word problem given.

11.   When the scarecrow gets his brain in the movie *Wizard of Oz*, he immediately says: "the sum of the square roots of any two sides of an isosceles triangle is equal to the square root of the remaining side." Is the scarecrow right? If not, what should he have said and why?

12.   What are the two sides of a right triangle that meet at a right angle called?

13.   What is the longest side of a right triangle called?

14.   If the three angles of a triangle are 20°, 40°, and 120°, is it a right triangle?

15.   The sum of the three angles in a triangle is 180°. An equiangular triangle has all three angles equal. Can a right triangle be equiangular?

Section 5.3 Pythagorean Theorem: Finding the Length of the Hypotenuse
Practice Problems 5.3
For Problem 1-6, find the hypotenuse of the triangle given and round your solution to the nearest tenth.
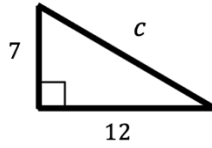
1.

2.
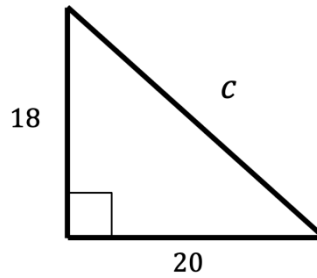
3.
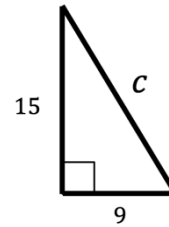
4. $a = 10$ cm.
    $b = 9$ cm.
    $c = ?$

5. $a = 5$ in.
    $b = 14$ in.
    $c = ?$

6. $a = 5.2$
    $b = 3.9$
    $c = ?$

For Problem 7-12, solve the word problem given.

7.   Find the length of the diagonal of a rectangle that is 11 cm. long and 7 cm. wide. Label the drawing to help you solve it. Round your solution to the nearest hundredth centimeter.
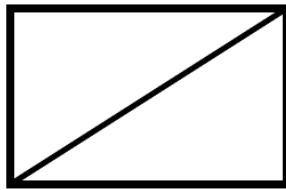
8.   What is the length of the diagonal of a square with a side length of 4 cm.?

For Problem 9-12, use the given information and diagram to solve the problem.

Mike wants to know how far it is across the lake at its widest part. However, it is so large that there is no way to run any type of device across the water to measure it. He decides to use indirect measurement. He will place a pole on one side of the lake at A and place another pole on the other side of the lake at B. He will place a third pole at C so that it makes a right angle with sides AC and BC.
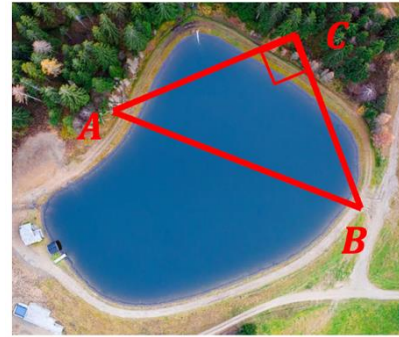
9.   How could Mike use this information and the Pythagorean Theorem to find the length across the lake?

10.   If it is 80 feet from pole A to pole C and 95 feet from pole B to pole C, what is the distance across the lake from pole A to pole B? Round your solution to the nearest tenth of a foot.

11.   If the distance from pole B to pole C is actually 75 feet instead of 95 feet, how does that change the distance from pole A to pole B?

12.   If Mike knows the length from pole A to pole B is 80 feet and the length from pole B to pole C is 75 feet, would 90 feet of bridge be enough to lay across the widest part of the lake (pole A to pole B)?

Section 5.4 Pythagorean Theorem: Finding the Leg

Practice Problems 5.4

For Problem 1-6, find the length of the missing leg in the triangle given. Round your solution to the nearest tenths place.
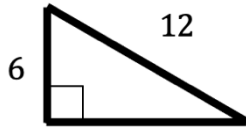
1.   2.   3.
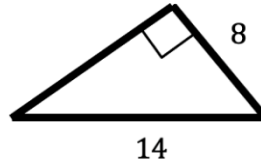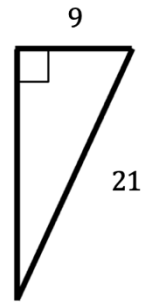
4. $a = 5$ 5. $a = 6$ 6. $a = 13$
        $b = ?$ $b = ?$ $b = ?$
        $c = 11$ $c = 14$ $c = 22$

For Problem 7-12, solve the word problem given.

7.   Find the perimeter of a triangle that has a hypotenuse of 13 cm. and a leg of 12 cm. (Remember, perimeter is the length around the figure.)

8.   Find the area of a right triangle with a hypotenuse of 17 in. and a leg of 8 in. (Remember, area is how many square units are inside the figure.)

9.	If a right triangle has an area of 48 sq. ft. and its shortest leg is 8 ft., find the length of its longest leg. Round your solution to the nearest tenth of a foot.

10.	What is the length of the hypotenuse of the triangle in Problem 6? Round your solution to the nearest tenth of a foot.

11.	If an isosceles right triangle has a leg that is 14 inches long, what is the length of its hypotenuse?

12.	Find the side length of a square that has a diagonal length of 17 cm. Round your solution to the nearest hundredth of a centimeter.

Section 5.5 Finding the Length of Any Missing Side of a Right Triangle
Practice Problems 5.5
For Problem 1-4, use the information and diagram below to solve the problem. Write your solution in exact form.

Before you found the length of sides of triangles by surrounding the triangle with a square and finding the area of the square. Now, you can find the length of a segment on a dot grid by surrounding the segment with a right triangle and using the Pythagorean Theorem to find the length.
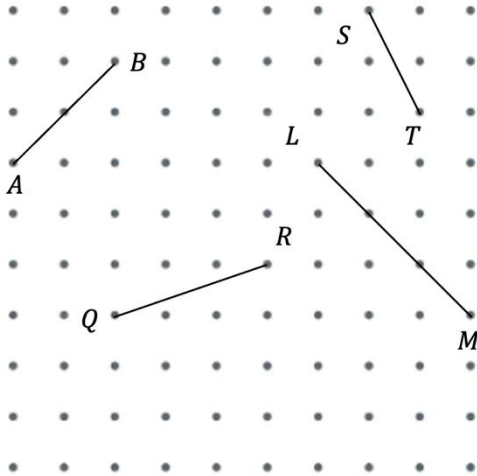
1.   Find the distance of AB using the Pythagorean Theorem.

2.   Find the distance of LM using the Pythagorean Theorem.

3.   Find the distance of QR using the Pythagorean Theorem.

4.   Find the distance of ST using the Pythagorean Theorem.

For Problem 5-10, use the triangles below to solve the problem.
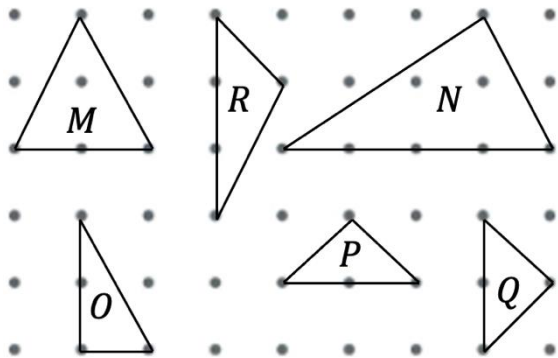
5.   Circle the right triangle(s) and label the legs a and b and the hypotenuse c. Put a square symbol at the right angle.

6.   Explain why the other triangles in Problem 5 that are not circled are not right triangles.

7.   Find the length of the diagonal side of O using the Pythagorean Theorem.

8.   Find the length of the side of triangle P by making two right triangles and using the Pythagorean Theorem.

9.   Find the length of the left side of triangle N by drawing a line from the top vertex straight down to the other side to make a large and small right triangle. Use the Pythagorean Theorem for the larger triangle on the left.

10.   Find the slanted side lengths of M by splitting it into two right triangles.

<u>Section 5.6 Pythagorean Triples</u>
<u>Practice Problems 5.6</u>
For Problem 1-7, tell whether the triangles are right triangles.

1. $3, 8, 14$ 2. $15, 20, 25$

3. $4, 7, 9$ 4. $9, 9, 15$

5. $16, 30, 34$ 6. $1, 2, 5$

7. $8, 8, 16$

For Problem 8-12, solve the word problem given.

8.   Which problems in Problem 1-7 are isosceles right triangles?

9.   How do you know which number is the hypotenuse in Problem 1-7?

10.   What is the perimeter of the largest right triangle in Problem 1-7?

11.   Given $a = 3$ ft., $b = 5$ ft., and $c = 7$ ft., is the triangle formed by these side lengths a right triangle?

12.   A baseball diamond has 90 feet between each base and the lines form right angles at the bases. Using the Pythagorean Theorem, find approximately how many feet it is from home plate to second base. (Draw a diagram to help you solve the problem.)

Section 5.7 The Pythagorean Theorem and the Distance Formula
Practice Problems 5.7

For Problem 1- 4, graph and connect the points given to make the hypotenuse of a right triangle. Draw the right triangle and answer the following questions.

a) What is the length of the horizontal leg?
b) What is the length of the vertical leg?
c) What is the length of the hypotenuse? (Use the Pythagorean Theorem equation to answer this one.)

1. $(-2, -4)$ and $(-4, 1)$
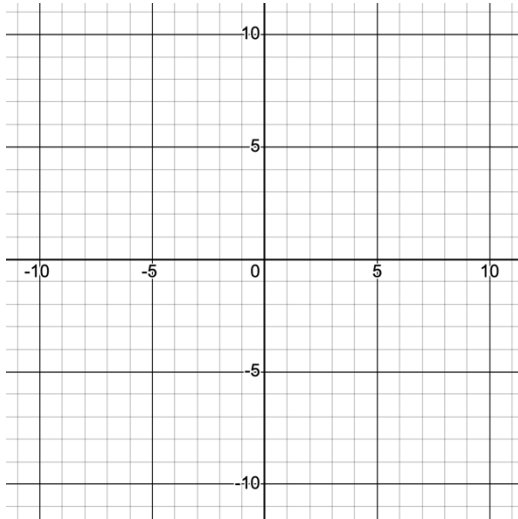
2. $(1, 1)$ and $(2, 3)$
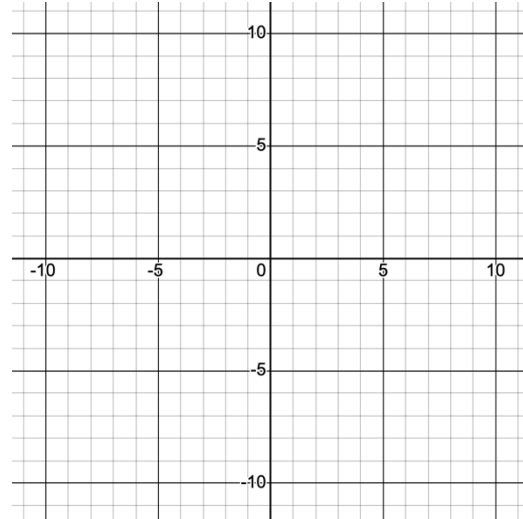
3. $(-3, -3)$ and $(7, 4)$

4. $(4, 2)$ and $(6, -3)$

11
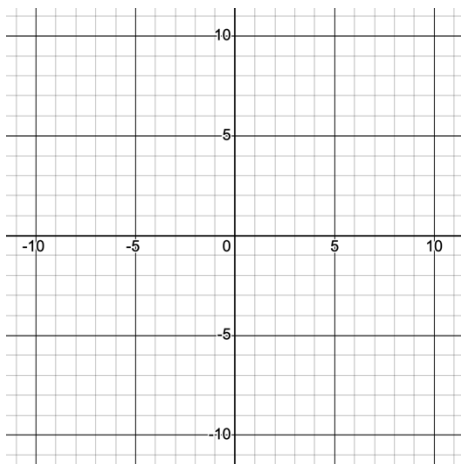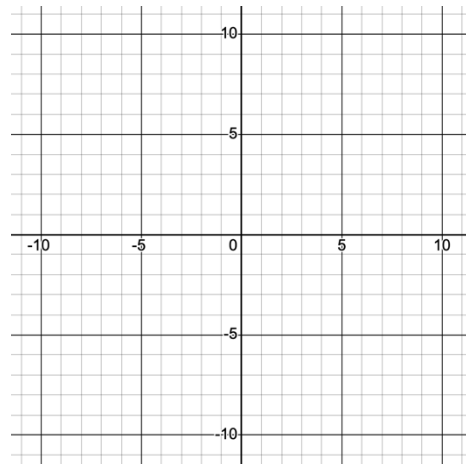
For Problem 5-8, use $|x_2 - x_1|$ to find the horizontal distance between the two points given. Check your solution with a) of Problem 1- 4.

5. $(-2, -4)$ and $(-4, 1)$ 6. $(1, 1)$ and $(2, 3)$

7. $(-3, -3)$ and $(7, 4)$ 8. $(4, 2)$ and $(6, -3)$

For Problem 9 -12, use $|y_2 - y_1|$ to find the vertical distance between the two points given. Check your solution with b) of Problem 1- 4.

9. $(2, -4)$ and $(-4, 1)$ 10. $(1, 1)$ and $(2, 3)$

11. $(-3, -3)$ and $(7, 4)$ 12. $(4, 2)$ and $(6, -3)$

Challenge Problem:
Find the lengths of each side and then find the perimeter of the triangle below. Give your solution in decimal form rounded to the nearest tenth. (Hint: This is not a right triangle. Form right triangles on the outside of the sides of the original triangle making each side of the triangles the hypotenuse of the new ones you formed.) Check your solution using the distance formula.
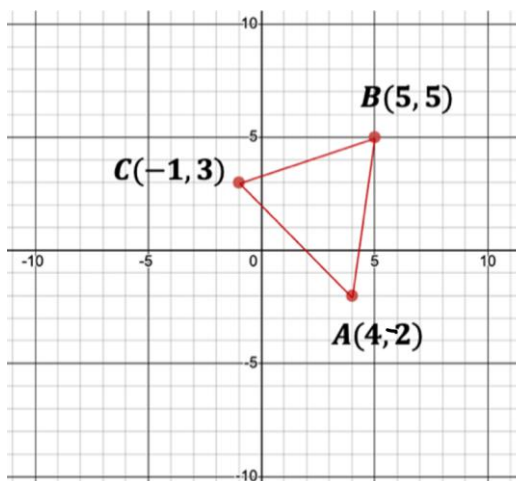
Section 5.8 Pythagorean Theorem Word Problems
Practice Problems 5.8
For Problem 1-10, solve the word problem given.

1.　　A ladder is placed against a wall. The ladder is 20 feet long and is placed 6 feet from the wall. How far up the wall will the ladder reach?
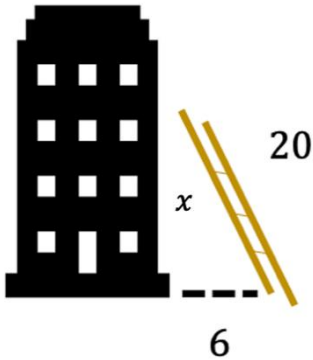
2.　　The diagonal of a computer screen is 17"; the width is 10". Find the length of the computer screen.
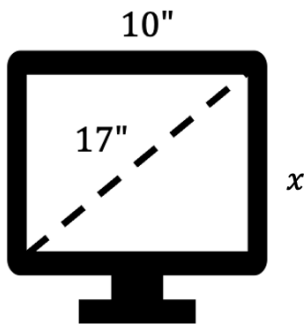
3.　　All but one tether of a hot air balloon comes loose and the balloon rises as far as it can, which is 25 feet off the ground and 15 feet to the west of where it started. What is the length of the last tether that is keeping the balloon from floating away?
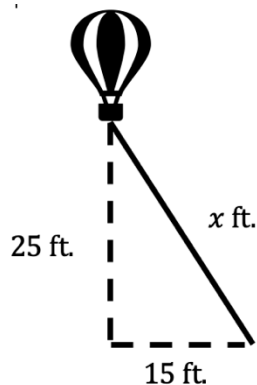
4.   The city council is building a skateboard ramp for the park. The base is 30 feet long and 5 feet high; how long is the ramp?

5 ft.

x ft.

30 ft.
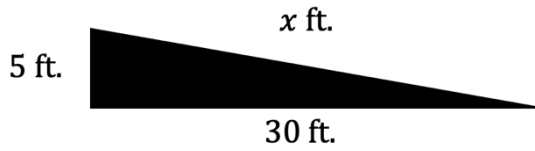
For Problem 5 and 6, use the given information to solve the problem.
A telephone pole is supported by four wires, which are in pairs of two, with each pair of wires being the same length on either side. The lower pair is secured 15 feet above the ground and the upper pair is secured 25 feet above the ground. All the wires are hooked in the ground 12 feet to the right and left of the pole.

5.   What is the length of the lower wires?   6.   What is the length of the upper wires?

25'
y   *y*
15'
x   *x*
12'   12'
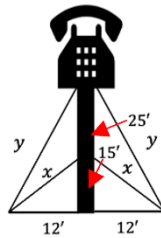
7.   A treehouse is 13 feet above the ground. The steps that go up to it are 22 feet from the treehouse along the diagonal. How far is the first step from the base of the tree?

22 ft.

13 ft.

x ft.
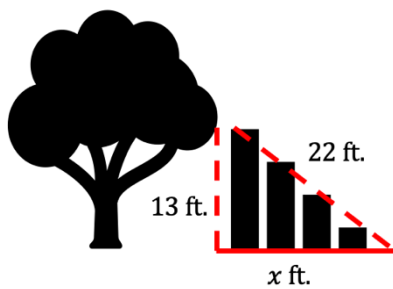
8.   A square table has a diagonal of 43 inches. What are the lengths of the sides of the table?

43 in. x

x
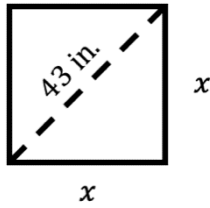
9.   Elizabeth and George are bicycling in the park. They are 50 yards from the river when they run into a barricade, so they are forced to take the long way. They go 30 yards due west, then turn to go due north. How far north will they have to go to reach the river?

50 yds.

x yds.

30 yds.
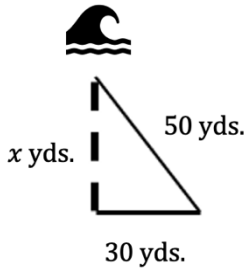
10.   A unit square is one unit wide by one unit long. How long is the diagonal from one corner to the opposite corner of the square?

1

1
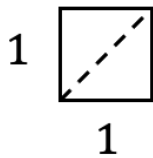
Section 5.9 The Wheel of Theodorus
Practice Problems 5.9
Follow the directions in the Lesson Notes to make your own Wheel of Theodorus. Create your own design around it and color it in. This is your own Theodorus Wheel of Art!
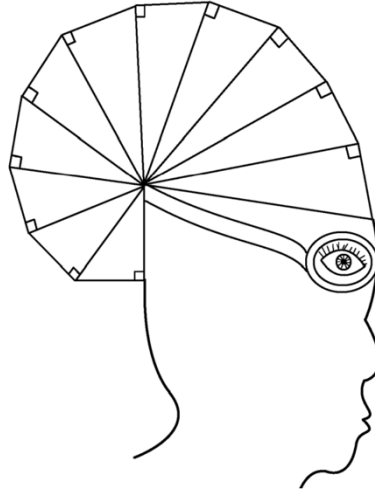
Theodorus of Cyrene was a Greek mathematician of antiquity. All we know about him has been passed down to us through Plato's dialogues (Plato was a Greek philosopher of antiquity). Some philosophers and historians believe Theodorus is a fictional (not real) character created by Plato. Some believe he was very real and worked on many mathematical concepts that moved the subject forward. Either way, what we do have left and what Plato credits to Theodorus is the Wheel of Theodorus.

What Plato tells us about Theodorus of Cyrene is that he proved all the square roots of non-square integers from 3 to 17 are irrational. Plato believed Theodorus stopped at 17 because the hypotenuse of $\sqrt{17}$ is part of the last triangle that does not overlap the Wheel. In short, the Wheel of Theodorus is a spiral of right triangles placed edge to edge to create a shape pleasing shape.

The Wheel of Theodorus can also be called the Einstein Spiral (as in Albert Einstein, a mathematician I am sure you have heard of). We do not know exactly why this is so.

There is much mystery surrounding the Wheel of Theodorus; however, we know the mathematics of it, and math does not change over time, we just learn more about it. Likewise, our God does not change. In James 1:7 He tells us: "For I the Lord do not change…"

In Section 5.11 Practice Problems, we will see how the Wheel of Theodorus relates to the Pythagorean Theorem.

Section 5.10 Deriving the Pythagorean Theorem using Algebra and Geometry
Practice Problems 5.10
For Problem 1-9, use the given information to fill in the table.

Problem	Side Lengths a	Side Lengths b	c	Does $a^2 + b^2 = c^2$?	Is it a Right Triangle?
1.	3	4	5		
2.	6	8	10		
3.	5	12	13		
4.	5	6	10		
5.	7	24	25		
6.	4	4	4		
7.	8	15	17		
8.	1	2	2		
9.	10	24	26		

For Problem 10-12, use the table/diagram given to solve the problem.

10.   Any of the three sides that satisfy the Pythagorean Theorem are called a Pythagorean Triple. Some triples are dilations of old ones. They are just doubled or tripled, etc. The list below is Pythagorean triples. Circle the ones that are new ones and not dilations of those in Problem 1-9. These are called Primitive Pythagorean Triples.

Side Lengths		
a	b	c
15	20	25
20	21	29
16	30	34
18	24	30

11.   There is an old city sewage system that goes along a river. The new sewage system is being routed outside the city away from the river. How much longer will the new sewage system travel than it did before?

Old Sewage System

City

28 miles

96 miles

New Sewage System
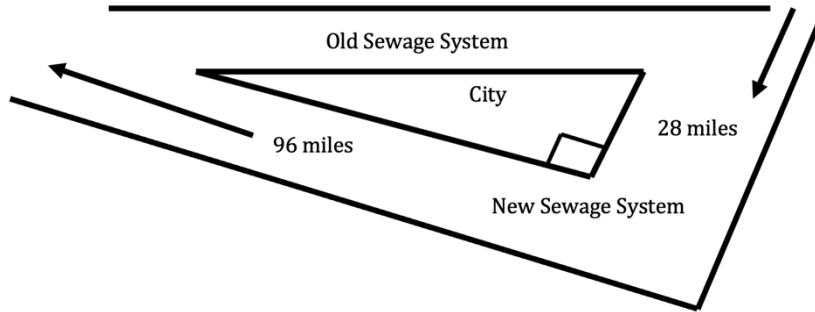
12.   Use the Pythagorean Theorem to find the lengths of the hypotenuse of the triangle and use the chop strategy to find the area of c^2 in terms of a number. Each square is equal to 1 square unit. Between dots horizontally or vertically is one unit.

c^2

b^2

c

b

a

a^2
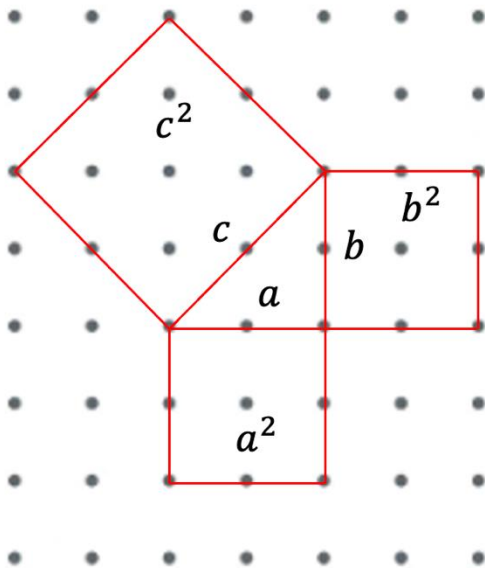
Section 5.11 The Pythagorean Theorem and the Golden Rectangle
Practice Problems 5.11

In Section 5.9, you constructed the Wheel of Theodorus. In this section, you will see how the Pythagorean Theorem relates to the Wheel of Theodorus.

Theodorus lived in the 5th century and was born in Cyrene (northern Africa). Later in his life, he taught in Cyrene as well as Athens. He would go on to be the main philosopher at the Cyrene school of moral philosophy and a member of the society of Pythagoras. Theodorus tutored Plato and also spent much time with Socrates; now, both are famous philosophers of Antiquity.

Theodorus believed the joys and pains of life were not good or bad. What was important, he thought, was to be cheerful and wise. To keep these would make one sufficiently happy.

Many of the findings of Pythagoras' students were often attributed to Pythagoras. (It was said that if you contradicted Pythagoras, you could lose your life.) The discovery that $\sqrt{2}$ is an irrational number is an example of a finding being attributed to Pythagoras.

Clearly, Theodorus did work with irrational numbers because Plato wrote about him in his work, *Theostatus*. Part of his writing included Theodorus proving the side (root) of a square with three square units and five square units was not commeasurable with the unit length; the ratio did not seem to be a rational number. This must have been an interesting finding because the Pythagorean Brotherhood believed that numbers were pleasing to the gods.
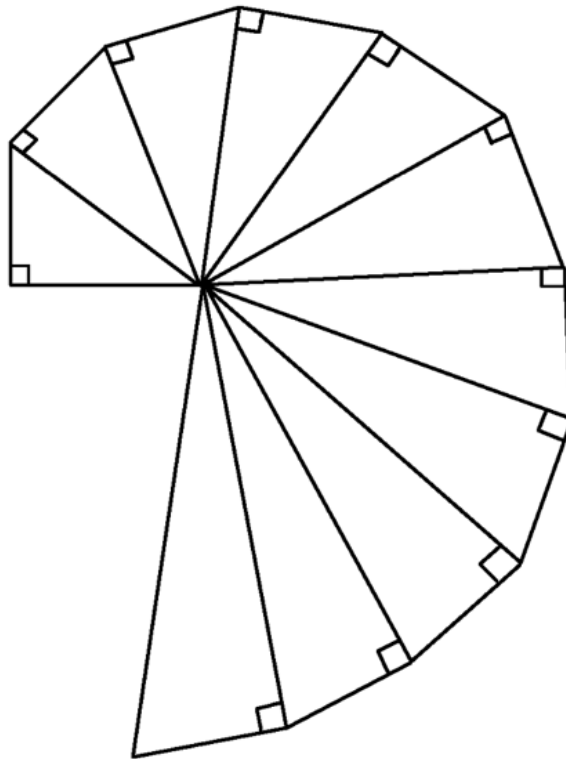
Measure each hypotenuse with a ruler. Put the measurements in the second column of the table. Write it also as a decimal approximation. Use the Pythagorean Theorem and a calculator to find the square root of the hypotenuse in exact form. Put this finding in the third column of the table. Simplify perfect square root numbers. Find the decimal approximation for this exact square root in the third column using a calculator. Round it to the ten-thousandths place and put this in the fourth column of the table.

What do you notice about the numbers in the second column and fourth column?

Triangle	Measurement on Ruler	Square Root Hypotenuse	Decimal Approximation on Calculator
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			

Section 5.12 Pythagorean Theorem and the Fibonacci Numbers

Practice Problems 5.12

1.   Use the Fibonacci numbers $2, 3, 5, 8$ to generate a Pythagorean triangle. Check your solution.

2.   Edouard Lucas lived in the 1800s. He generated a series of numbers: $1, 3, 4, 7, 11, 18$ ... and are called Lucas numbers. They are related to Fibonacci numbers as each number is the sum of the previous two numbers and Lucas is said to have assigned the name Fibonacci numbers to $1, 1, 2, 3, 5$ ... in honor of Leonardo of Pisa.

$$F_{n+1} = F_{n-1} + F_n \text{ if } n > 1$$

$$F_0 = 0$$

$$F_1 = 1$$

This means $F_2 = F_{1+1}$ and $n = 1$

$$F_{1+1} = F_{1-1} + F_1$$

$$F_2 = F_0 + F_1$$

F_2 is the second number in the series (1)

$$F_0 = 0 \text{ and } F_1 = 1$$

$0 + 1 = 1$ (Therefore, this works)

Lucas said that $F(n)^2 + F(n + 1)^2 = F(2n + 1)$

Let $n = 1$ and see if it works. Remember, $F(3)$ means the third number in the series: $1, 1, 2, 3, 5, 8, 13$ ... so $F(3) = 2$. It does not refer to the number 3 in the series.

3.   The golden rectangles drawn earlier have sides of Fibonacci numbers: $1, 1, 2, 3, 5$ ... The squares of these are the areas. Find the area of the squares of the golden rectangle for the first nine Fibonacci numbers.

4.   Find a pattern in the triangle below. Based on the pattern, what two numbers are the factors that go in the box?

$$1^2 + 1^2 = 1 \times 2$$
$$1^2 + 1^2 + 2^2 = 2 \times 3$$
$$1^2 + 1^2 + 2^2 + 3^2 = 3 \times 5$$
$$1^2 + 1^2 + 2^2 + 3^2 + 5^2 = 5 \times 8$$

$$1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 = \boxed{}$$

5.   Below is another method to find Pythagorean triangles using fractions. Complete Step 3-5.

   1.   Take two odd numbers that differ by two (use 5 and 7)

   2.   Make them into unit fractions ($\frac{1}{5}$ and $\frac{1}{7}$)

   3.   Find the sum of the two fractions.

            The numerator and denominator in the sum are two legs of a right triangle.

   4.   Use the Pythagorean Theorem to find the hypotenuse

   5.   Write the three sides as a Pythagorean Triple

6.   Does the same method in Problem 5 work for two even numbers that differ by two? Try it with 6 and 8 and see. Show your work.

7.   Of birthdays, let the number of the month and the number of the day be the first two numbers of the Fibonacci Sequence. Using the birthday of May 15 (5 and 15), find the next seven numbers in the series. (The next number is 20).

8.   Divide each number by the previous number and write the ratio as a decimal approximation. What number does the ratio approach?

9.   Use your birthday to start the Fibonacci Sequence. List the next seven numbers in the series.

10.   Divide each number by the previous number and write the ratio as a decimal approximation. What number does the ratio approach?

Section 5.13 More Pythagorean Theorem Problems
Practice Problems 5.13
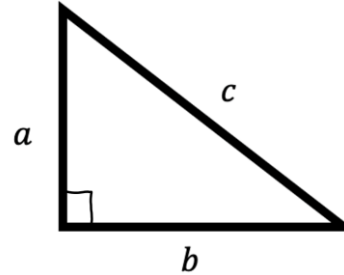For Problem 1-4 use the diagram to find the unknown side. Round any decimal solution to the tenths place.

1. $a = 5$ $b = 7$

2. $a = 10$ $b = 24$
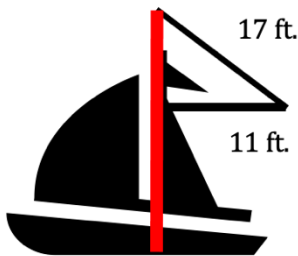
3. $a = 2$ $c = 8$

4. $b = 7$ $c = 13$

For Problem 5-10, use the information and/or diagram given to solve the problem.

5.   The sailboat pictured below has a top sail with 11 feet along the bottom and 17 feet along the slant side of the sail. If the mast is 36 feet from the bottom of the boat to the bottom of the top sail, what is the height of the entire mast?

6.   A sailor wants to coat his top sail with waterproofing materials. One quart of these materials covers 60 square feet of sail. How many gallons does he need to waterproof the sail from Problem 5? (There are 4 quarts in 1 gallon.)

7.   A pool stick manufacturer has boxes 4 meters long by 3.3 meters wide. What is the longest pool stick he could manufacture to put in the box to be shipped?
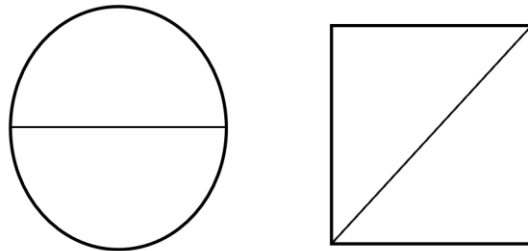
8.   Find the length of the side of the trapezoid. The top base is 4 inches long and the bottom base is 6 inches long.
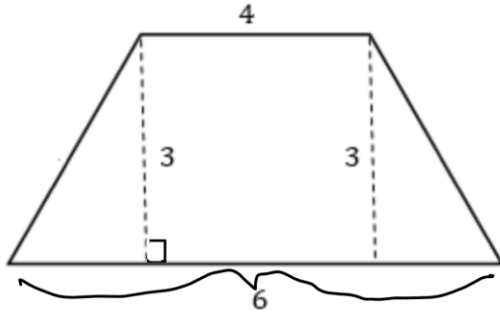
9.   Find the length of the edge *e* that is bolded in black from the top of the pyramid to the corner of the base.
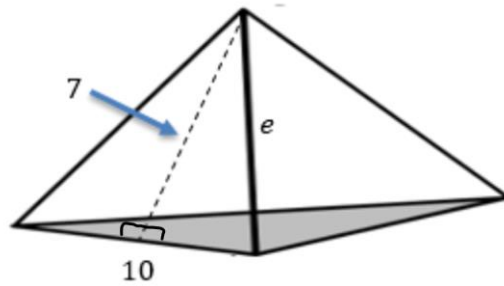
10.   Why are manhole covers over sewers round instead of square?

<u>Section 5.14 Module Review</u>

For Problem 1-4, write Yes or No for whether the side lengths given are the sides of a right triangle.

1. $3, 4, 8$ 2. $6, 8, 10$

3. $17, 18, 26$ 4. $10, 24, 26$

For Problem 5 and 6, tell whether the triangle is right or not and explain how you know.

5.

6.

For Problem 7-9, use the given diagram to solve the problem.

7. A student builds the Wheel of Theodorus with a triangle that is 1.5" for each leg. About how long is the hypotenuse?

8.   If the student builds a second triangle onto the triangle from Problem 7, about how long will the next hypotenuse be?
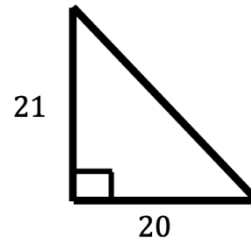
9.   Continuing the process from Problem 8, how long will the next hypotenuse be?

For Problem 10-15, find the missing side length in the right triangle given and approximate the decimal solution to the nearest tenths place.
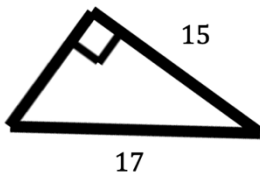
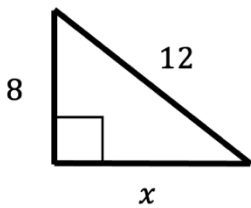10.   11.

25

15

21

20

12.

15

17
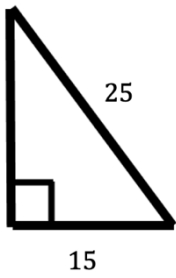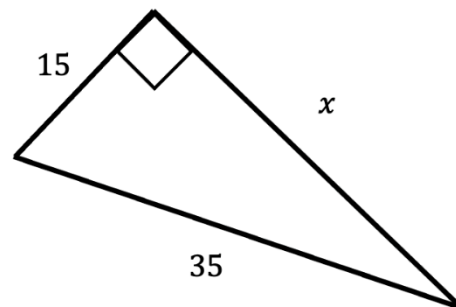
For Problem 13-15, find the missing side in the right triangle and approximate the decimal solution to the nearest tenths place.

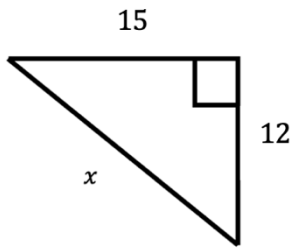13.   14.

12

8

x

15

x

35

15.

15

12

x

For Problem 16 and 17, tell whether the side lengths given are the legs and hypotenuse of a right triangle.

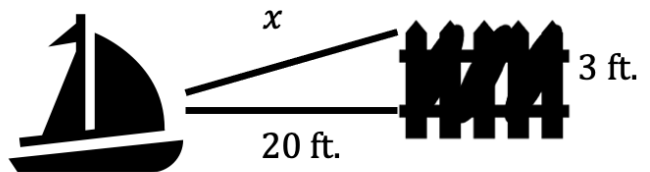16. $7, 9, 15$ 17. $10, 13, 18$

For Problem 18-22, solve the word problem given.
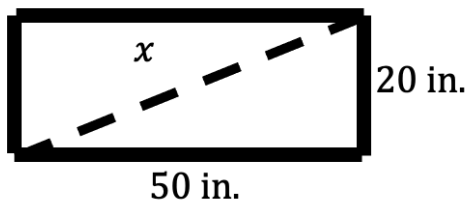
18.   Find the distance between the points:

a) $(3, 7)$ and $(-5, -5)$ b) $(0,0)$ and $(12, 9)$

19.   A boat is tied to the dock at a marina. One of its lines breaks loose. The dock is 3 feet above the boat and the line that is still attached and securing the boat is 20 feet long. How far is the boat from the dock?

x

3 ft.

20 ft.

20.   Find the diagonal of a rectangle that has a width of 20 in. and a length of 50 in.

x

20 in.

50 in.

21.   The mast of a boat is 40 ft. tall. It is secured to the front sail of the boat by a line that is 60 ft. long. How far is the line from the base of the mast to the front base of the front sail given they meet at a right angle?

60 ft.

40 ft.

x
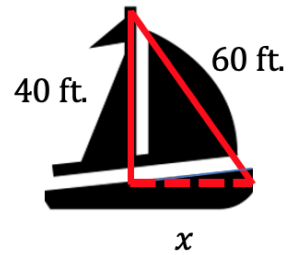
22.   Angie's kite is flying really high today. She has let out 130 ft. of string and is standing 70 ft. from where the kite is perpendicular to the ground. she is standing off the ground and to the right below the kite. How high is the kite from the ground? Approximate the height to the nearest tenths place.
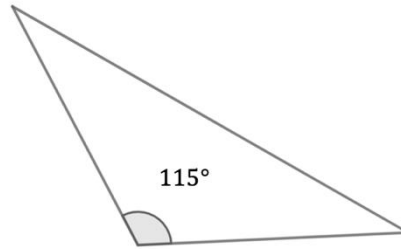
It is a moon kite!

130 ft.

x

70 ft.
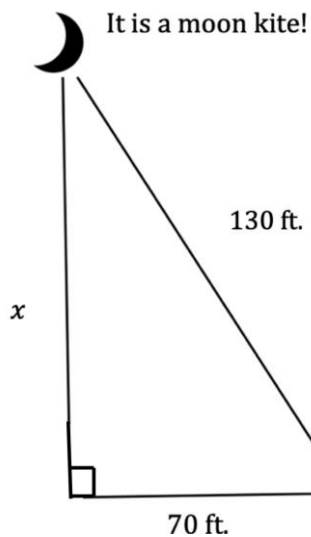
Section 5.15 Module Test
For Problem 1-3, tell if the triangle given is acute, obtuse, or right.

1.   2.

3.

80°

53°   47°
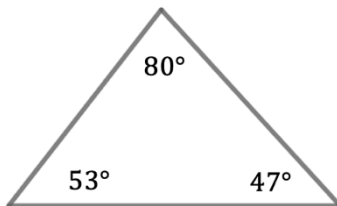
For Problem 4-9, tell whether the numbers given are Pythagorean Triples.

4. $6, 8, 16$ 5. $6, 8, 12$

6. $20, 48, 52$ 7. $3, 4, 6$

8. $4, 6, 8$ 9. $9, 12, 15$

For Problem 10-12, solve the word problem given.

10.   Two friends are walking through a wood. One friend travels southwest for 3 miles. The other friend travels southeast. They travel as a 90° angle. When the first friend stops at 3 miles, they are 5 miles apart. How far has the second friend traveled?

11.   A tent is anchored on either side by a rope 9 m. long from the top of the center pole. The staked rope on either side is 6 m. from the pole in the center of the tent. How high is the center pole of tent? Approximate the decimal solution to the nearest tenth of a meter.
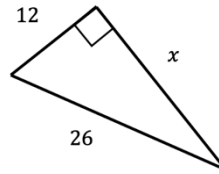
12.   If one leg of a right triangle is 24 inches and the hypotenuse is 36 inches, how long is the other leg?

For Problem 13-15, find the missing side length. (Approximate decimals to the nearest tenth.)
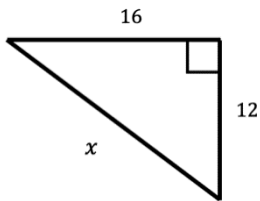
13.

10
18
x

14.

12
x
26

15.

16
12
x
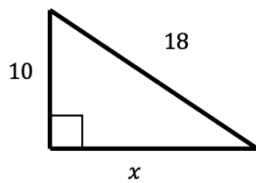
For Problem 16 and 17, tell whether the side lengths given make up a right triangle.

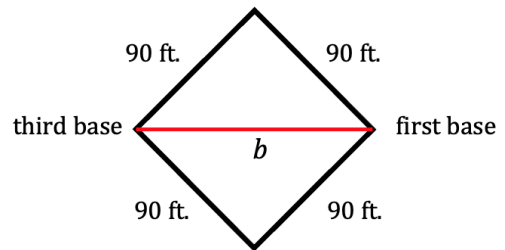16. $15, 20, 25$ 17. $13, 14, 15$

31

For Problem 18, use the distance formula to find the distance between the points given. (Round any decimal solutions to the nearest tenths place.)

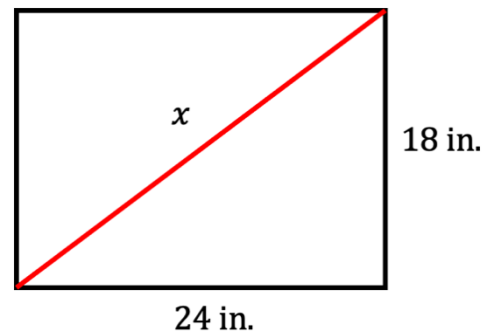a) $(3, 9)$ and $(-8, -5)$ b) $(0, 0)$ and $(16, 20)$

For Problem 19-22, solve the word problem given.

19.   A baseball diamond is a perfect square with sides of 90 ft. How far is it from first base straight across the field to third base?

20.   Find the length of the diagonal of a rectangle with a width of 18 in. and a length of 24 in.

21.   Seth is playing soccer. He runs from where he is 17 feet diagonally to the ball and with a great kick, scores a goal. Seth ends up 8 feet south of where he started. How far is Seth up the field from where he started (west)?
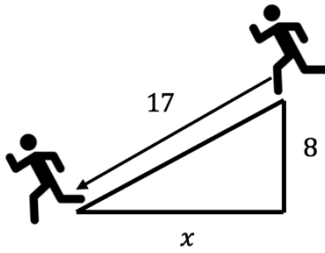
22.   Joel built a table that is 36 in. by 48 in. He wants to know if the corners are square, so he measures the diagonal and gets 60 in. Are the corners square?