Module 8 Exponential Functions

Section 8.1 Exploring Exponential Equations

Practice Problems 8.1

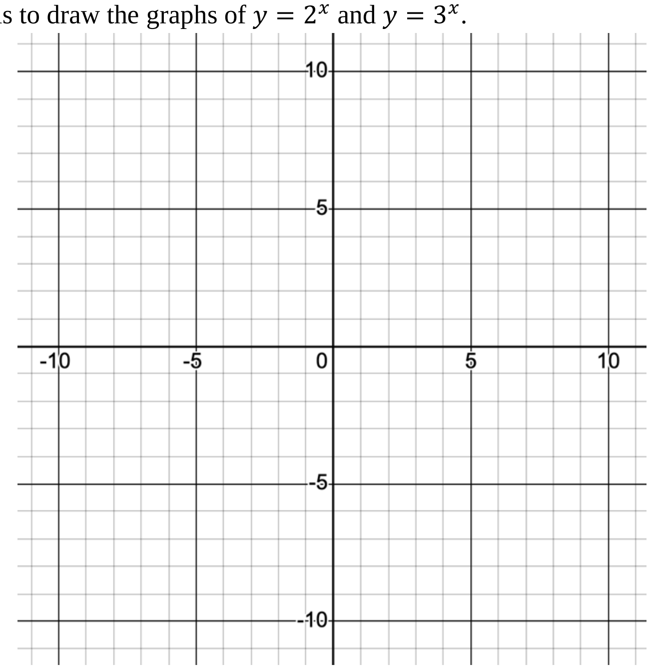
For Problem 1-4, use the exponential equations $y = 2^x$ and $y = 3^x$ to solve the problem.

1. In $y = 2^x$ and $y = 3^x$, which is growing at a faster rate? How do you know?

2. Complete the table for $y = 2^x$ and $y = 3^x$.

x	2^x	3^x
-3		
-2		
-1		
0		
1		
2		
3		

3. Use colored pencils to draw the graphs of $y = 2^x$ and $y = 3^x$.



4. Which graph is steeper? Why? What does the steeper graph represent?

For Problem 5-10, use the information from Problem 1-4 and the exponential equations $y = 2^{-x}$ and $y = 3^{-x}$ to solve the problem.

5. a) If $3^2 = 9$, what is 3^{-2}? b) If $5^{-3} = \frac{1}{125}$, what is 5^3?

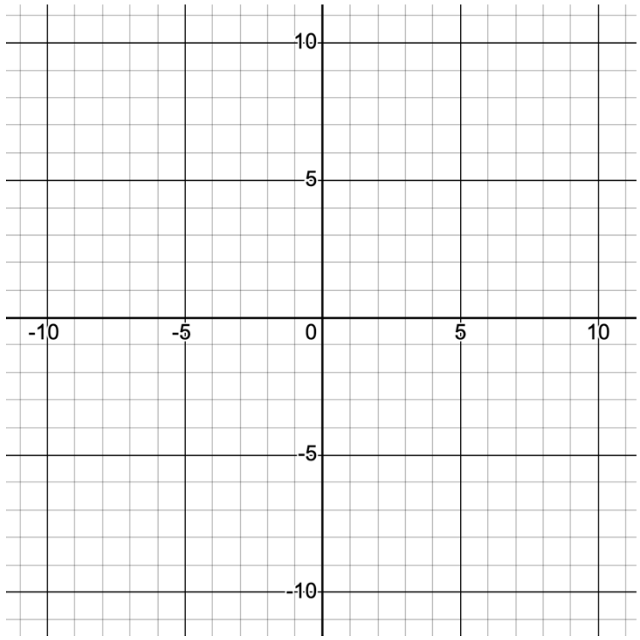
6. How are the solutions to 3^2 and 3^{-2} related?

7. Why is 0^0 undefined?

8. Using what you learned about patterns in Problem 5, complete the table for $y = 2^{-x}$ and $y = 3^{-x}$. (Look back at the table in Problem 2 for help.)

$\boldsymbol{x}$	$\mathbf{2^{-x}}$	$\mathbf{3^{-x}}$
-3		
-2		
-1		
0		
1		
2		
3		

9. Use two different colored pencils to draw the graphs of $y = 2^{-x}$ and $y = 3^{-x}$.



10. How do the graphs of $y = 2^{-x}$ and $y = 3^{-x}$ compare to the graphs of $y = 2^{x}$ and $y = 3^{x}$?

For Problem 11-15, rewrite the exponential expressions as fractions.

11. y^{-3}

12. x^{-4}

13. $-5x^{-2}$

14. $2x^{-3}$

15. $-\frac{1}{4}y^{5}$

For Problem 16-20, simplify the fractional expression given.

16. $\frac{8^5}{8^7}$

17. $\frac{18x^3}{2x^3}$

18. $\frac{x^2}{x^{-3}}$

19. $\frac{3x^2yz^3}{4xyz}$

20. $\frac{x^{-5}}{x^{-2}}$

Section 8.2 Investigating Exponential Bases

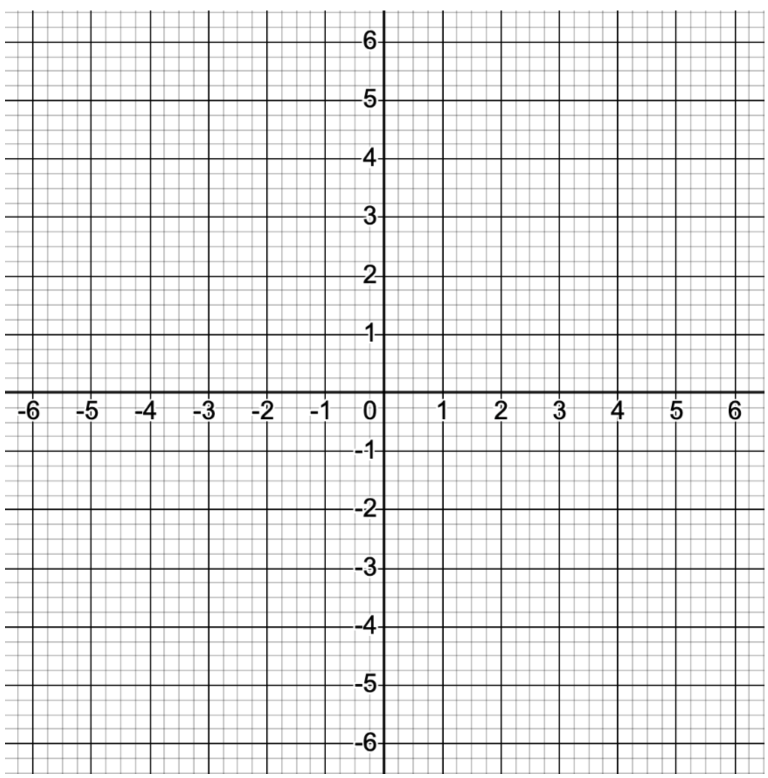
Practice Problems 8.2

For Problem 1-6, use the information and diagram given to solve the problem.

1. Let $b = 0$ and complete the table for $y = b^x$ ($y = 0^x$) using integer values from -3 to 3. Is this an exponential function?

x	y
-3	
-2	
-1	
0	
1	
2	
3	

2. What happens to the graph of $y = 0^x$ when $x > 0$? Sketch the graph of $y = 0^x$.



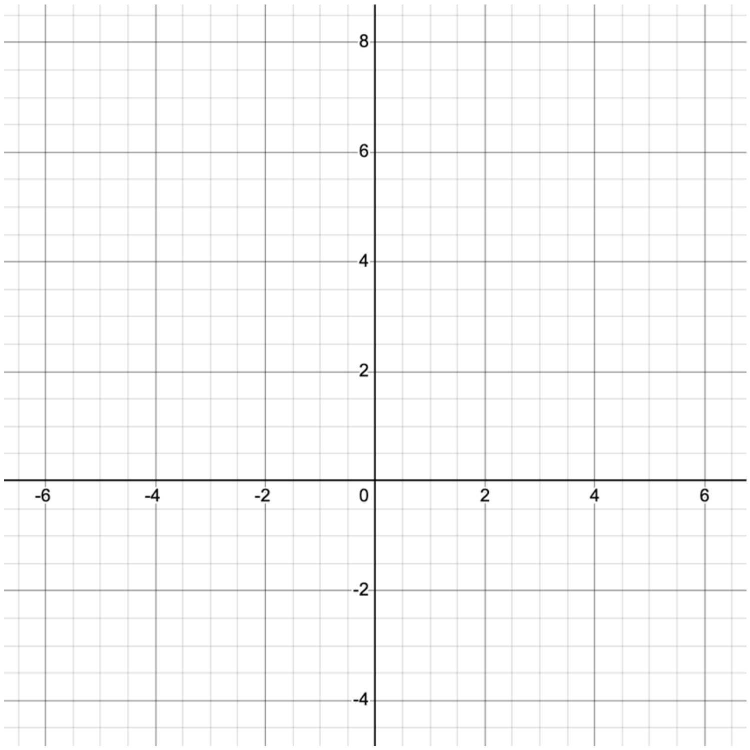
3. Complete the table below for the base of $\frac{1}{2}$ in the equation $y = (\frac{1}{2})^x$.

x	y
-3	
-2	
-1	
0	
1	
2	
3	

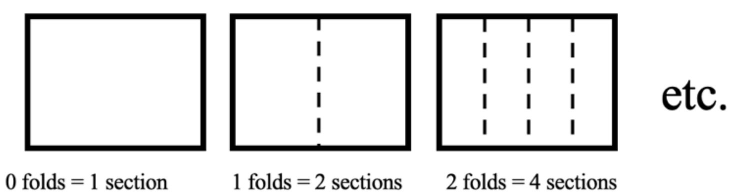
4. Complete the table below for the base of 0.2 in the equation $y = (0.2)^x$.

x	y
-3	
-2	
-1	
0	
1	
2	
3	

5. Draw the graphs of $y = (\frac{1}{2})^x$ and $y = (0.2)^x$ on the graph below using different colored pencils. What is similar about the graphs? What is different?



6. Take a piece of paper and fold it in half. Count the number of sections made by the fold and record them in the table below. Fold that half in half again so there are two folds parallel to each other (all going in the same direction). Unfold it, count the sections and record them in the table below. Fold it back over the folds and then fold it again making a third fold. Repeat this process until there are six folds in the paper. (Remember to record your sections in the table.)



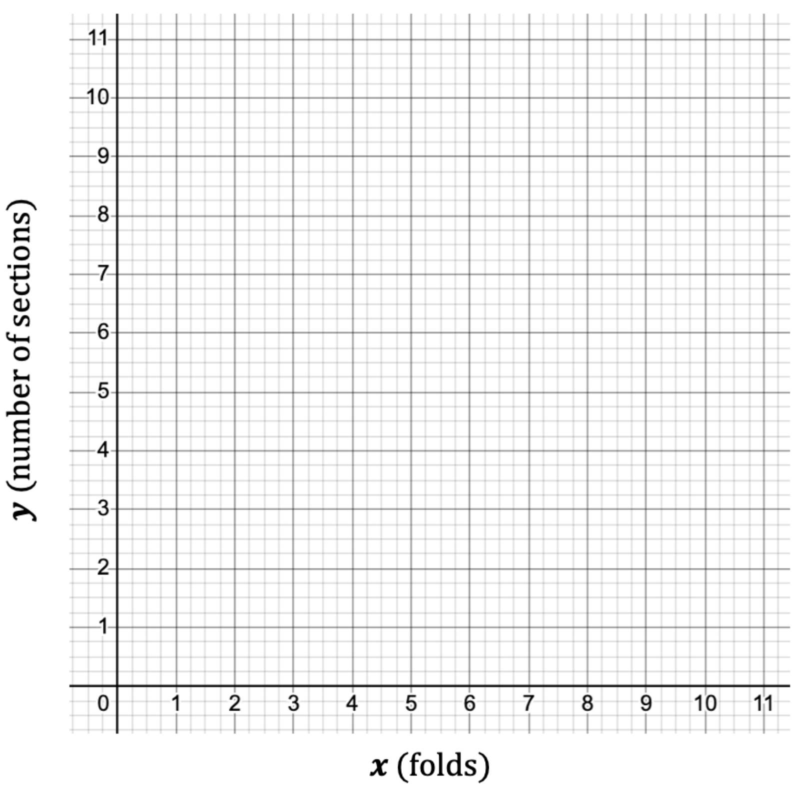
x (folds)	y (number of sections)
0	
1	
2	
3	
4	
5	
6	

a) What happens to the layers of the piece of paper each time you fold it?

b) Write an equation to represent the process. Let x be folds and y be the number of sections.

c) Estimate how many folds it takes to make the number of sections equal to the pages in a traditional math book (about 600 pages).

d) Draw the graph from the table. Is it exponential?

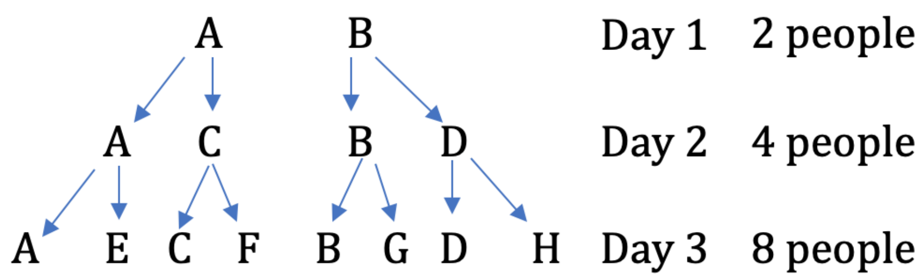


For Problem 7-10, use the given situation to solve the problem.
At the end of the day, two friends are talking and share that they heard from someone: “God is alive.” The next day, both friends tell one friend each: “God is alive.” The third day, each friend that heard “God is alive” the second day tells a friend: “God is alive,” and so on and so on.

Fill in the blanks:

7. The first day, two people heard: “God is alive.” By the end of the second day, __________ people have heard: “God is alive.” By the end of the third day, __________ people have heard: “God is alive.”

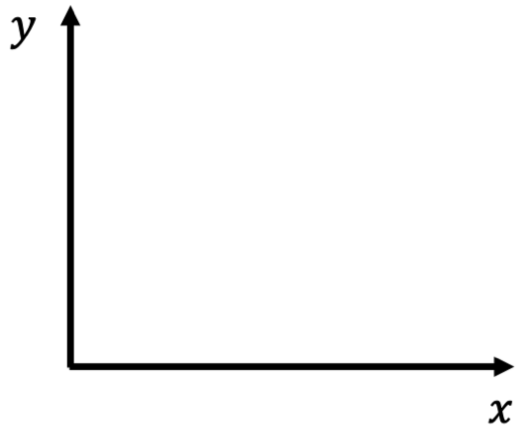
8. How many friends have heard: “God is alive” by the fifth day? Make a tree diagram; do you see a pattern?



9. Complete the table and graph for “God is alive!” What type of the function does it model? Can you find the rule so you can get an equation in terms of x (days) and y (number of people who have heard: “God is alive!”).

x	y
1	
2	
3	
4	
5	
6	

$y =$ ______________



10. By the end of which day will 95 people have heard: “God is alive?”

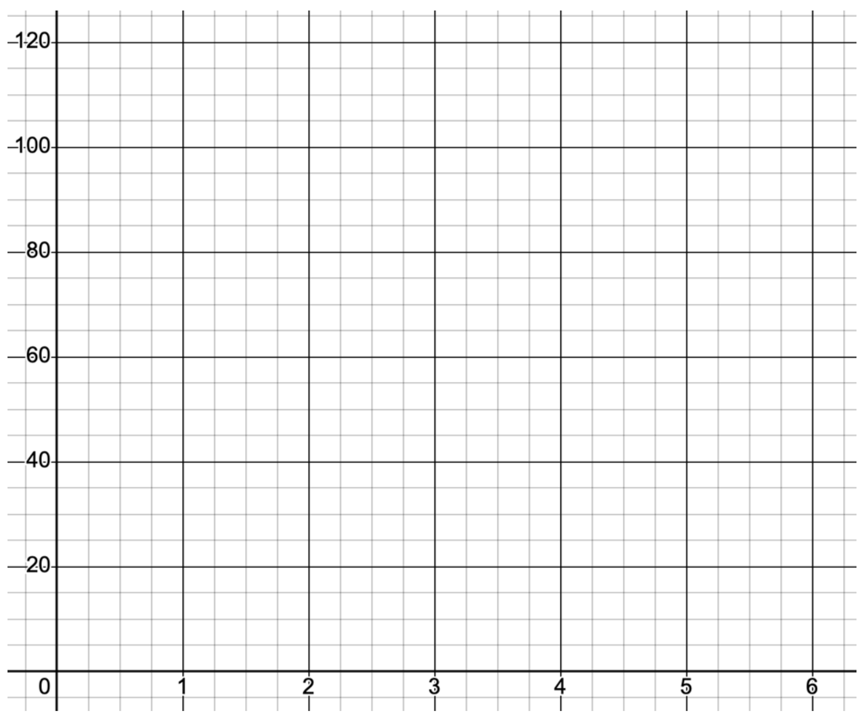
Section 8.3 Geometric Sequences

Practice Problems 8.3

For Problem 1-9, use the information and/or diagram given to solve the problem.

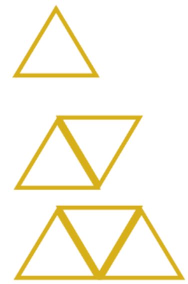
1. The Good News problem seems to generate the same table as the Paper Folding problem. What is the equation for the Good News problem? Let x be the day and y be the number of people who have heard the Good News. How is the equation for the Good News problem the same as the equation for the Paper Folding problem? How is it different?

2. Using the table for the Good News problem, draw the graph below. Is it linear or exponential?



3. Let us make triangles from toothpicks. It takes 3 toothpicks to make 1 triangle.
Adding 2 more toothpicks to the triangle makes 2 triangles and a total of 5 toothpicks.
Adding 2 more toothpicks to the triangles makes 3 triangles and a total of 7 toothpicks.
Keep adding toothpicks to make a total of 7 triangles.
Complete the table below for the total number of toothpicks after each triangle is made.

x (triangles)	y (toothpicks)
1	
2	
3	
4	
5	
6	
7	



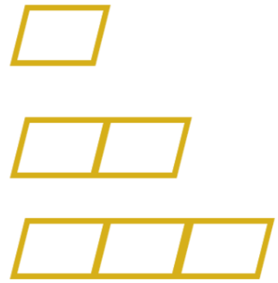
4. Is the Toothpick problem an arithmetic sequence or a geometric sequence? Does it have a common difference or a common ratio? What is the common difference or common ratio?

5. If a common ratio comes from taking each total number (term value) and dividing it by the previous total number (term value), where does a common difference come from? Use the table from the Looking Back section to explain why.

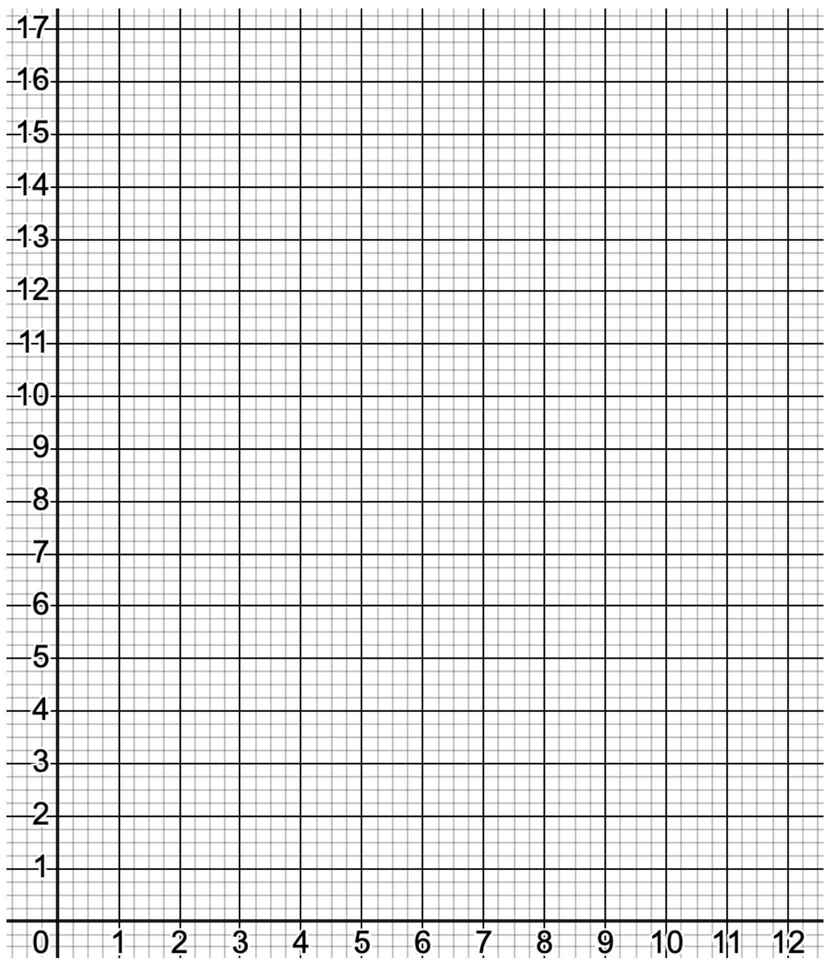
n (term number)	$a(n)$ (term value)
1	8
2	12
3	16
4	20
5	24
6	28

6. Follow the instructions from Problem 3 but rather than making triangles, make parallelograms with toothpicks. Complete the table below.

x (parallelograms)	y (toothpicks)
1	
2	
3	
4	
5	
6	
7	



7. Draw the graph for the Toothpick and Parallelogram problems.



8. Are the Toothpick and Parallelogram problems arithmetic sequences or geometric sequences? What are the common differences or common ratios?

9. Are the Toothpick and Parallelogram problems linear or exponential?

For Problem 10-15, find the common ratio and the next three terms of the geometric sequence given.

10. $3, 9, 27, 51,$ ______________, ______________, ______________

11. $27, 9, 3, 1,$ ______________, ______________, ______________

12. $\frac{1}{4}, \frac{1}{2}, 1, 2,$ ______________, ______________, ______________

13. $0.3, 0.06, 0.012,$ ______________, ______________, ______________

14. $2, 8, 32, 128,$ ______________, ______________, ______________

15. $5, 25, 125, 625,$ ______________, ______________, ______________

For Problem 16-20, use the given table to solve the problem.

x (Term Number)	y (Term Value)
1	3
2	6
3	
4	
5	
6	

16. Assuming the sequence in the table is arithmetic, complete the table for y when the common difference is $6 - 3$.

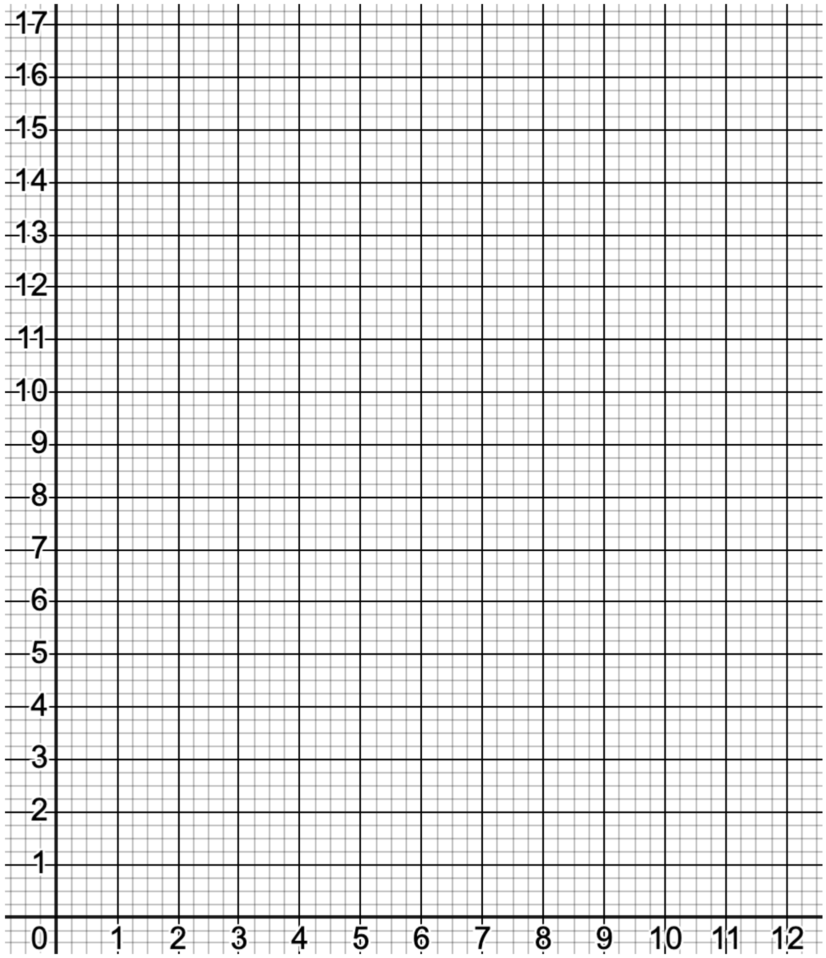
17. What is being added each time?

18. Write the value of the terms of the sequence from the table. What are the next three terms in the sequence?

________, ________, ________, ________, ________, ________

1 2 3 4 5 6

19. Draw a graph of the arithmetic sequence.



20. Write the recursive formula for the arithmetic sequence.

Section 8.4 Recursive Formulas for Geometric Sequences

Practice Problems 8.4

For Problem 1-10, identify the common ratio in the geometric sequence given and write the recursive formula for the sequence.

1. $3, 12, 48, 192, \ldots$

2. $-32, -16, -8, -4, -2, \ldots$

3. $100, 50, 25, 12.5, \ldots$

4. $3, 9, 27, 81, \ldots$

5. $14, 42, 126, 378, \ldots$

6. $\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{8}{3}, \ldots$

7. $\frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \frac{1}{256}$

8. $2, -4, 8, -16, \ldots$

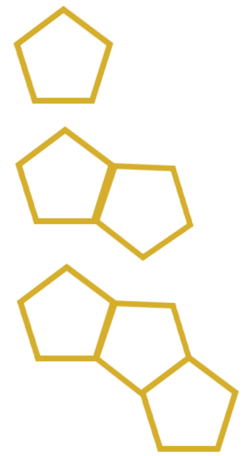
9. $3, -9, 27, -81, \ldots$

10. $5, 25, 125, 625, \ldots$

For Problem 11-15, use the information and/or diagram to solve the problem given.

11. Another toothpick problem uses toothpicks to make pentagons. The adjacent toothpicks (that form the shared side of the pentagon) represent 1 toothpick. Complete the table below and answer the questions.

x (pentagons)	y (toothpicks)
1	5
2	9
3	
4	
5	
6	
7	



a) Is this an arithmetic sequence or a geometric sequence? Why?

b) Without drawing it, is the graph of this data linear or exponential?

12. The recursive formula for the Toothpick problem with triangles is $a(n) = a(n-1) + 2$. The recursive formula for the Toothpick problem with parallelograms is $a(n) = a(n-1) + 3$. What is the recursive formula for the Toothpick problem with pentagons? What pattern do you notice? Where does it come from? What would the recursive formula be for the Toothpick problem using hexagons (a six-sided polygons)?

13. In the Toothpick problem, if polygons are arranged in a row and only the number of outside toothpicks (those on the perimeter) are counted, not the total number of toothpicks (interiors are excluded), the problem changes. Complete the table below and answer the questions in Problem 14 and 15.

Number of Polygons	Outside (Exterior) Toothpicks			
	Triangle	Parallelogram	Pentagon	Hexagon
1	3	4	5	6
2	4	6	8	
3				
4				
5				
6				

14. What is the common difference for each polygon?

a) Triangle

b) Parallelogram

c) Pentagon

d) Hexagon

15. a) Write a recursive formula for outside (exterior) toothpicks to generate the sequence values for…

a triangle:

a parallelogram:

a pentagon:

a hexagon:

b) How many outside edges would be on each arrangement if it consisted of 7 tiles?

a triangle:

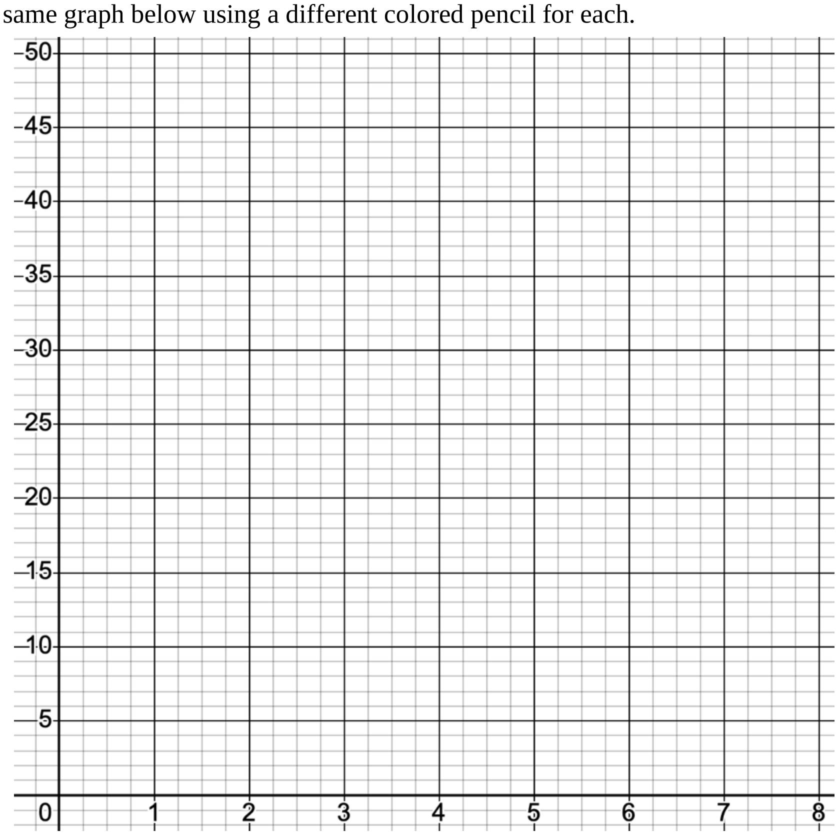
a parallelogram:

a pentagon:

a hexagon:

What pattern do you notice?

c) Let the x-axis be the number of polygons and the y-axis be the total number of outside edges. Plot the four shapes on the same graph below using a different colored pencil for each.



d) How do the graphs determine whether it is an arithmetic or geometric sequence?

e) Which is the steepest graph and why?

For Problem 16-20, use the given table to solve the problem.

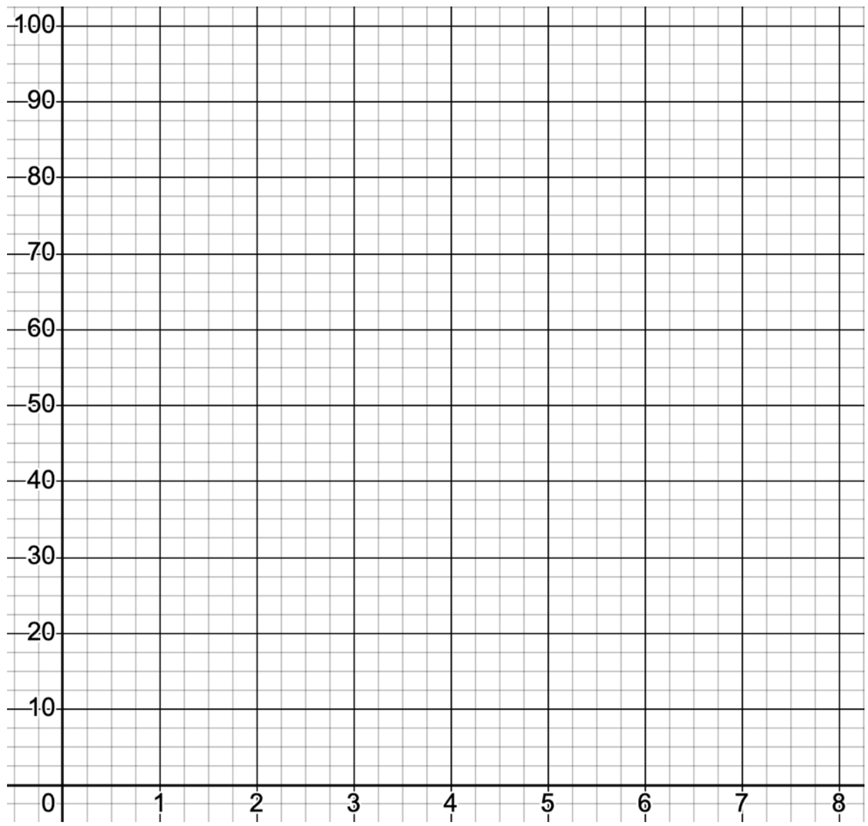
x (Term number)	y (Term value)
1	3
2	6
3	
4	
5	

16. Assuming the sequence in the table is geometric, complete the table for y when the common ratio is $6 \div 3$.

17. Write the value for the next four terms of the sequence when x is $6 - 9$.

18. What is being multiplied each time? What is the common ratio?

19. Draw the graph of the equation below.



20. Write the recursive formula for the geometric sequence.

Section 8.5 Explicit Formulas for Geometric Sequences

Practice Problems 8.5

For Problem 1-8, find the initial value and the common ratio and write the explicit formula for the sequence given.

1. $2, 2.6, 3.38, 4.394, \ldots$

2. $4, 16, 64, 256, \ldots$

3. $\frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ldots$

4. $7, 21, 63, 189, \ldots$

5. $4\,; 80;\ 1{,}600; 32{,}000$

6. $1, 2, 4, 8$

7. $3; 4.2; 5.88; 8.232$

8. $5, 15, 45, 105, \ldots$

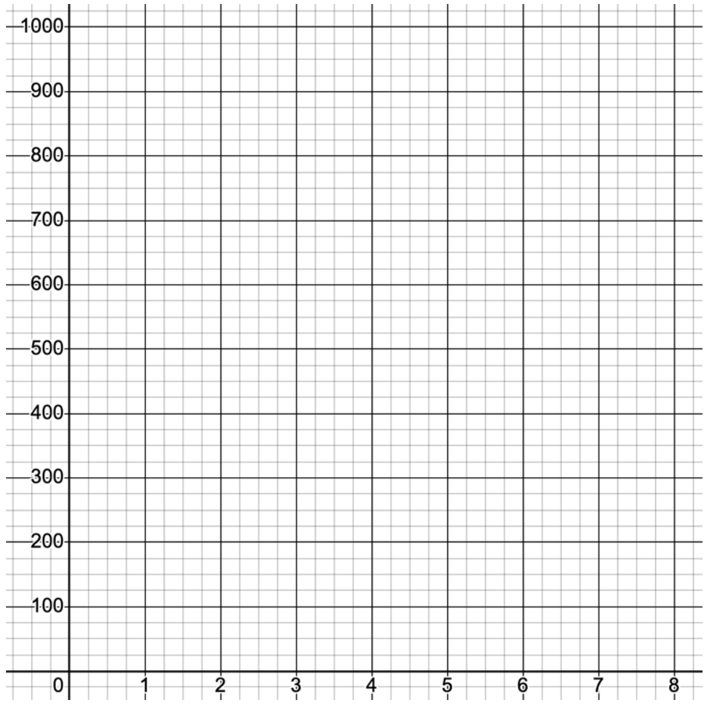
For Problem 9 and 10, solve the word problem given.

9. A geometric sequence has an initial value of 8 and a common ratio of 3.
   a) Write an explicit formula for the geometric sequence.

   b) List the first six terms of the sequence that go with the term number.

Term (x)	1	2	3	4	5	6
Term Value (y)						

c) Graph the function for the first six terms. Let x be the term number and y be the term value.



d) Is the sequence linear or exponential?

10. a) A yeast culture doubles in size every hour. If you put 8 grams of yeast culture in a dish, write an explicit formula for the growth. Use your explicit formula to find the culture size the 6^{th} hour.

b) What would be the explicit formula if the yeast culture from Problem 10 tripled every hour? Use this explicit formula to find the culture size the 6^{th} hour.

For Problem 11-20, use the given table to solve the problem.

x (term number)	y (term value)
1	3
2	6
3	12
4	24
5	48
6	96

11. Complete the sequence from the table: 3, __________, __________, __________, __________, __________

12. What is g_1? (Be careful here, the first term is represented by 1; it is the first term in the sequence in Problem 11.)

13. What is the common ratio, r?

14. What is r^{n-1} when n is 1?

15. What is the explicit formula for the geometric sequence? Use $g_n = g_1 r^{n-1}$.

16. Is the 7th term in the table g_6 or g_7?

17. Use the formula to find the 7th term.

18. What is the value of g_6?

19. What would you add to g_6 to get g_7?

20. What is the value of g_7?

Section 8.6 Exponential Growth

Practice Problems 8.6

Using the same steps from the previous Meemer bug growth experiment from the Lesson Notes, record the total population according to the table below. Only do 4 shakes. Then answer the questions given. This time, the growth rate will change, but the initial population will stay the same.

Meemer Bugs		
Color	**Growth Factor**	**Initial Population**
Yellow	For every yellow Meemer with the *m* side up or down, add 1 yellow Meemer	Start with 1 yellow Meemer
Orange	For every orange Meemer with the *m* side up or down, add 2 orange Meemers	Start with 1 orange Meemer
Red	For every red Meemer with the *m* side up or down, add 3 red Meemers	Start with 1 red Meemer

Stage of Growth	**Total Number of Yellow Meemers**	**Total Number of Orange Meemers**	**Total Number of Red Meemers**
0			
1			
2			
3			
4			

a) Describe the relationship between the total number of yellow Meemers and the shake number at Stage 1, 2, and 3. Describe the relationship between the total number of orange Meemers and the shake number at Stage 1, 2, and 3. Describe the relationship the total number of red Meemers and the shake number at Stage 1, 2, and 3.

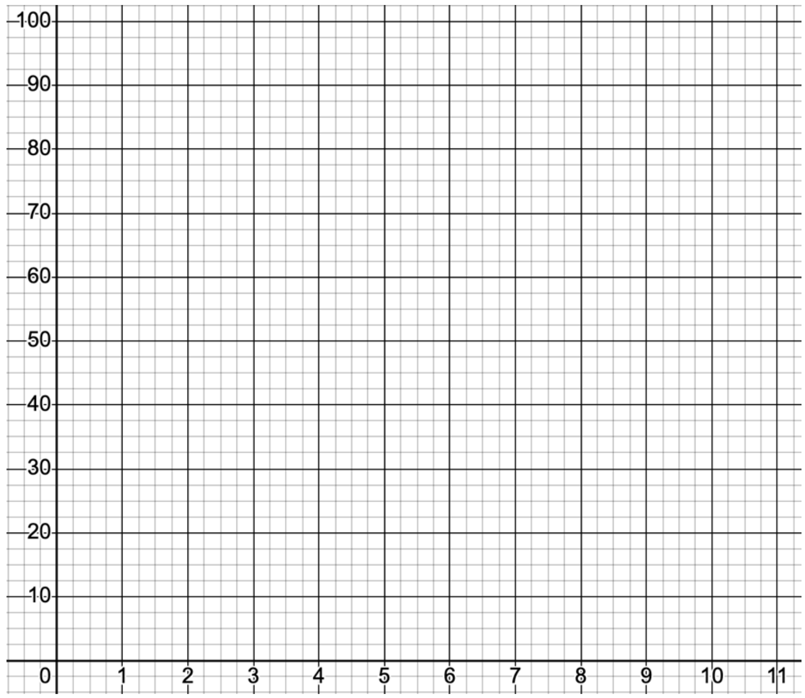
b) Write the relationship from a) as a mathematical equation. Let x represent the shake or Stage number and let y represent the number of Meemers.

c) What is the growth rate for:
   a. Orange Meemers?

   b. Red Meemers?

d) Which values represent the domain, and which represent the range?

e) Graph the equations using the Meemer colors. What type of equation and graph are these?



f) Is this a function? Are all exponential equations functions?

g) Which graph is the steepest and why?

Section 8.7 Exponential Decay

Practice Problems 8.7

For Problem 1-6, use the tables given to solve the problem.

1. Jocelyn is a lab technician. She has a petri dish of agar that helps bacteria grow. At each hour, the bacteria in the petri dish doubles. If Jocelyn begins the bacterial growth with 5 grams of bacteria, complete the table to show how many grams there will be after 7 hours.

Let $x =$ hours and $y =$ grams of bacteria.

x	y
0	
1	
2	
3	
4	
5	
6	
7	

2. Haley is a lab technician. She accidentally drops a chemical in a petri dish of agar with bacteria and every hour, half of the bacteria dissolves. If this petri dish had 66 grams of bacteria when Haley dropped the chemical in, how many grams of bacteria will there be after 7 hours? Complete the table to solve the problem.

x	y
0	
1	
2	
3	
4	
5	
6	
7	

3. Of the two equations $y = 5(2)^x$ and $y = 66(\frac{1}{2})^x$, which represents the situation in Problem 1 and which represents the situation in Problem 2? Explain why.

4. Demarco, another lab technician, cultured some bacteria that grows at a rate of 30% each day for a week. Today, there are 380 bacteria. How many bacteria will there be tomorrow?

5. Darvus, a fourth technician, works with cells that decay at a rate of 15% each day. If there are 250 cells today, how many will there be tomorrow?

6. Why can the exponential equation $y = 3(\frac{1}{2})^x$ also be written $y = 3(0.5)^x$? Is it an increasing or decreasing exponential?

For Problem 7-10, use the given situation to solve the problem.
Barry buys a car for $12,000; the car depreciates at 15% annually. The equation to represent the value of the car each year is $y = 12{,}000(0.85)^x$ in which x represents the number of years since the purchase of the car and y represents the total value of the car.

7. Why is the growth rate 0.85?

8. Will the value of the car be increasing or decreasing?

9. What will the car be worth in 3 years?

10. Approximately when will the car be worth half the amount Barry paid for it?

Section 8.8 The General Exponential Equation

Practice Problems 8.8

For Problem 1-4, use the equation from Example 1 of the Lesson Notes to solve the problem.

1. The same company, whose employees tripled each year, bought a computer for each new employee. Use the equation from Example 1 of the Lesson Notes to determine how many computers they will need to purchase by their fifth year of operation.

2. Is $y = 26(3)^x$ a recursive or explicit formula? Explain why.

3. Using the graph from Example 1 of the Lesson Notes, when will the company have approximately 200 employees?

4. The second year, the company decides to safety-check $\frac{1}{4}$ of the computers from each year on. How many computers will be refurbished in the 2^{nd} year?

For Problem 5-8, using the table given, find the values of a and b so that $y = ab^x$, then write the equation for the table.

5.

x	y
0	3
1	6
2	12
3	24
4	48

6.

x	y
0	4
1	2
2	1
3	0.5
4	0.25

7.

x	y
0	1
1	3
2	9
3	27
4	81

8.

x	y
0	$\frac{1}{2}$
1	$\frac{1}{4}$
2	$\frac{1}{8}$
3	$\frac{1}{16}$
4	$\frac{1}{32}$

For Problem 9-20, solve the word problem given.

9. In Problem 5-8, which are increasing exponentials? How do you know?

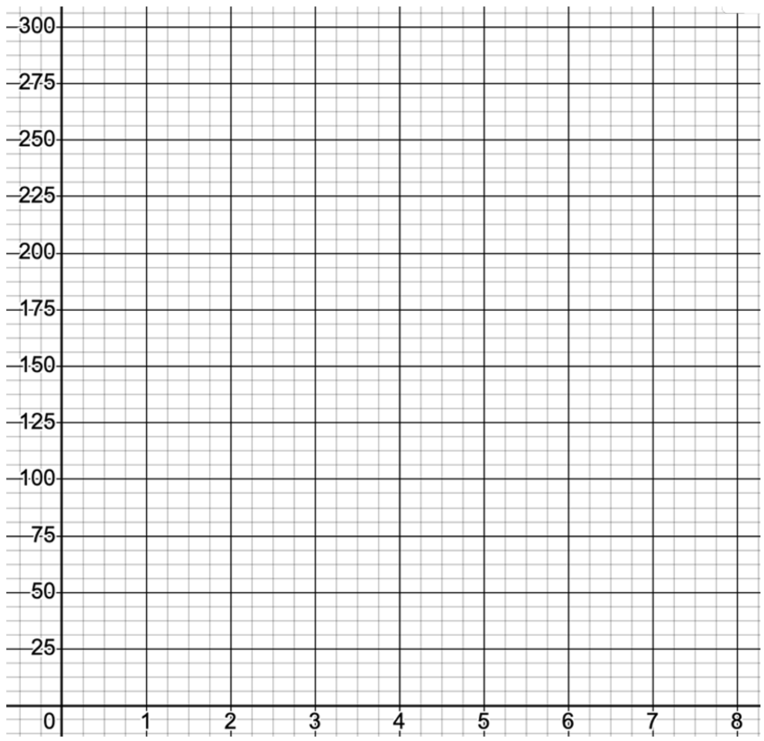
10. In Problem 5-8, which are decreasing exponentials? How do you know?

11. Complete the table that represents exponential growth.

x (term)	y (total)
0	4
1	8
2	16
3	
4	
5	
6	

12. For the table in Problem 11, find the rule in the form of $y = ab^x$. What is a? How do you know? What is b? How do you know?

13. Graph the table for Problem 11. What is the initial value from the table called?



14. If a bacterium grows weekly according to the geometric sequence $2, 6, 18, 54$ ... find the next three terms of the sequence.

15. In Problem 14, if 2 represents the initial number of bacteria, during what week is that?

16. Write the equation that represents the exponential growth of the bacteria from Problem 14.

17. How many bacteria will there be by the 10^{th} week?

18. What is the initial value in $y = 7 \cdot 2^x$?

19. What is the y-intercept in $y = 7 \cdot 2^x$?

20. What is the growth factor in $y = 7 \cdot 2^x$?

Section 8.9 Transformations of Exponential Equations

Practice Problems 8.9

For Problem 1-8, solve the word problem given.

1. Given the equation $y = 2^x$, how would the graph change for the equation $y = 2^x - 5$?

2. How would the values in the table of $y = 2^x$ change for the equation $y = 2^x - 5$?

3. Write the equation from the parent function of an exponential given the following constraints:
a) The initial value is -6
b) The growth factor is 3.2
c) The horizontal shift is left 1
d) The vertical shift is up 1

4. If the equation $c = 400(\frac{1}{2})^d$ represents cell division per hour, what do c and d represent?

5. In the equation $c = 400(\frac{1}{2})^d$, what do 400 and $\frac{1}{2}$ represent?

6. In the equation $c = 400(3)^d$ in which c is the total number of cells and d is each hour, is 3 a growth factor or decay factor?

7. In the equation $c = 400(3)^d$, how many cells will there be in four hours?

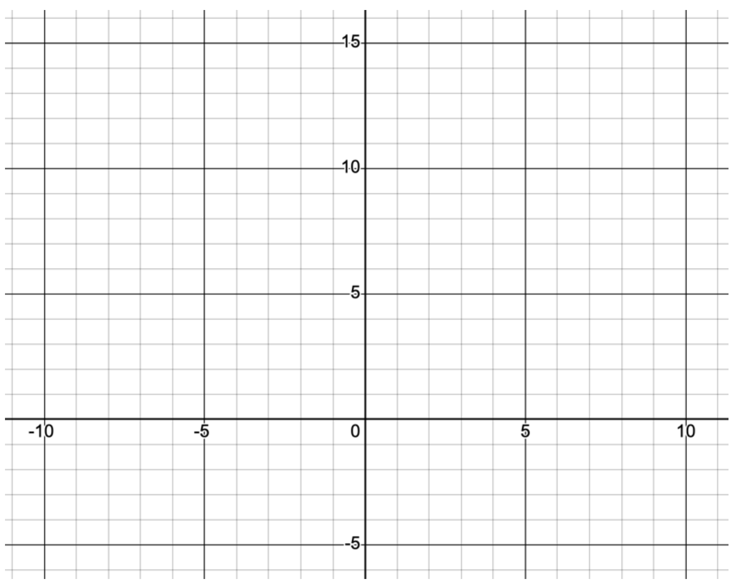
8. After a cell loses 75% of its viability every minute, how much viability does the cell have left? What would be the growth rate for the explicit formula $y = b^x$?

For Problem 9-20, use the tables/graphs given to solve the problems.

9. Below is the table for $y = 2^x$. Complete the table for $y = 2^x + 3$.

$\boldsymbol{x}$	$\mathbf{2^x}$	$\mathbf{2^x + 3}$
0	1	
1	2	
2	4	
3	8	
4	16	

10. Draw the graphs of $y = 2^x$ and $y = 2^x + 3$ in different colors. How are they similar? How are they different?



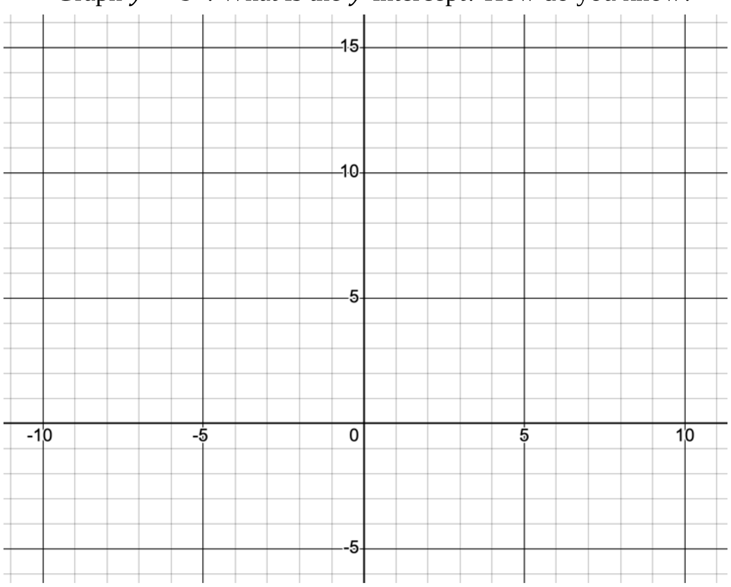
11. Is the graph of $y = 3^x$ more steep or less steep than $y = 2^x$? Why?

12. Is the graph of $y = (\frac{1}{3})^x$ more steep or less steep than $y = 2^x$? Why?

13. Complete the table for $y = 3^x$. What is the y-intercept? How do you know?

x	3^x
0	
1	
2	
3	
4	

14. Graph $y = 3^x$. What is the y-intercept? How do you know?



15. With a red colored pencil, sketch $y = 3^x + 4$ on the same graph as $y = 3^x$ in Problem 14. What is its y-intercept?

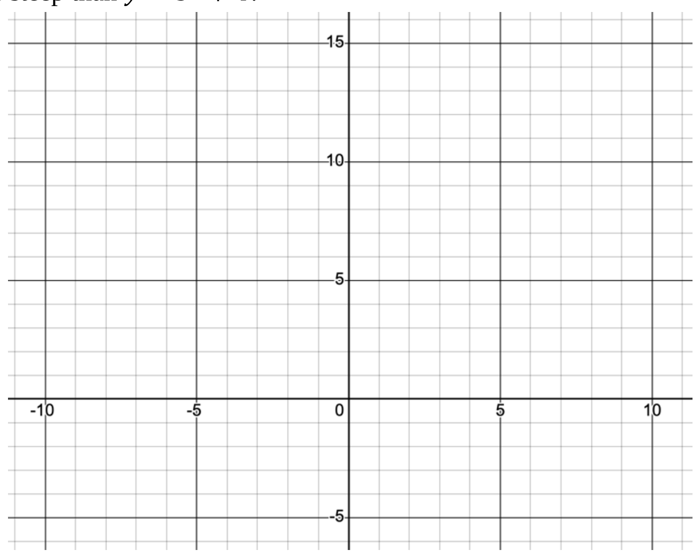
x	$3^x + 4$
0	
1	
2	
3	
4	

16. With a blue colored pencil, sketch $y = 3^x - 6$ on the same graph as $y = 3^x$ in Problem 14. What is its y-intercept?

x	$3^x - 6$
0	
1	
2	
3	
4	

17. With a red colored pencil, sketch a new graph of $y = 3^x + 4$. Use a green colored pencil to sketch $y = 5^x + 4$. Is $y = 5^x + 4$ more steep or less steep than $y = 3^x + 4$?

x	$5^x + 4$
0	
1	
2	
3	
4	



18. With a purple colored pencil, sketch the graph of $y = (\frac{1}{2})^x + 4$ on the same graph in Problem 17. What is different about the graph of $y = (\frac{1}{2})^x + 4$ from those of $y = 3^x + 4$ and $y = 5^x + 4$?

19. Is the graph of $y = (\frac{1}{2})^x + 4$ more steep or less steep than those of $y = 3^x + 4$ and $y = 5^x + 4$?

20. Are the graphs of $y = 3^x + 4$ and $y = 5^x + 4$ increasing or decreasing exponentials?

Section 8.10 Compound Interest

Practice Problems 8.10

For Problem 1-3, use Example 2 from the Lesson Notes to solve the problem.

1. Bobby used the method below to find Jordan's interest after four years. Complete the end balance for each year of the table. Does this also give $541.22 after four years?

Year	Starting Balance	Expanded Form	Exponential Form	End Balance
0	$500			$500
1	$500	$\$500(1+0.02)$	$\$500(1.02)^1$	
2	$500	$\$500(1+0.02)(1+0.02)$	$\$500(1.02)^2$	
3	$500	$\$500(1+0.02)(1+0.02)(1+0.02)$	$\$500(1.02)^3$	
4	$500	$\$500(1+0.02)(1+0.02)(1+0.02)(1+0.02)$	$\$500(1.02)^4$	

2. Joey used the equations below to find out how much each will save in 6, 7, and 10 years.

Jordan: $y = \$500(1.02)^x$

Adreyan: $y = \$300(1.04)^x$

Who will save more after 10 years?

3. How much more will Jordan have than Adreyan after 26 years of saving?

For Problem 4-10, solve the word problem given.

4. Kiera decides to be risky and invest in stocks. They decrease at a rate of 2% on her initial $4,000 investment.

a) What is her growth rate? (Hint: It is less than 100%, it is actually a decay rate)

b) What is the constant multiplier?

c) Write the exponential equation to model this situation.

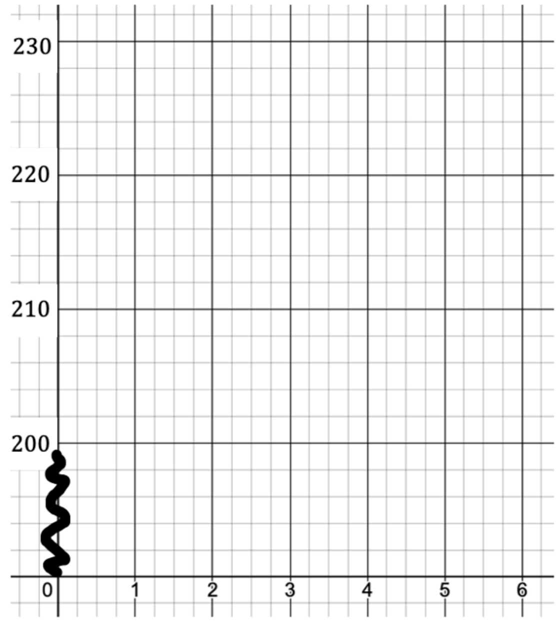
d) How much will she have left in her investment account after 4 years?

5. What is the exponential equation that models an initial investment of $3,400.00 with an annual interest rate of 0.8%?

6. Regina has $500. She needs $20 more by her next birthday. Should Regina invest at START Bank with an interest of 5% and a minimum deposit of $500 or split her money in two and put $250 in FUTURE Bank at an annual rate of 3% and $250 in GOLD Bank at an annual rate of 6% (GOLD Bank only allows a maximum deposit of $250 for their highest rate of 6%)?

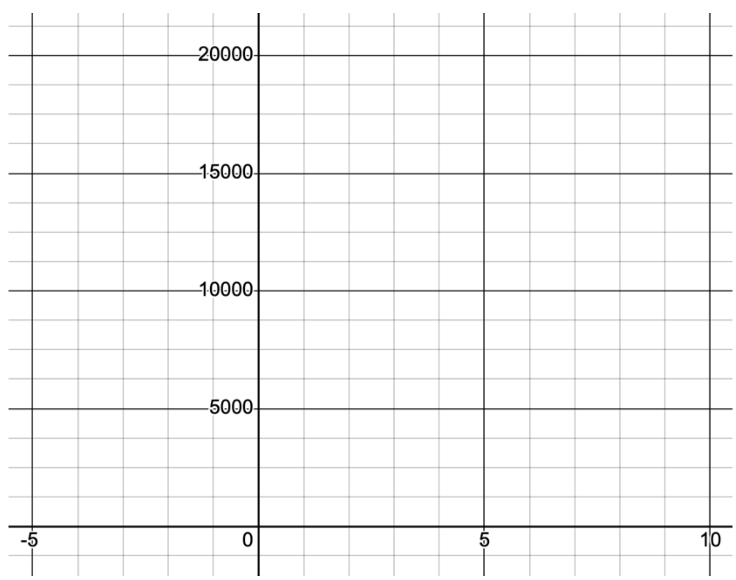
7. Amy invested \$200 in a bank that offers 5% interest compounded annually. Write an equation to model the total money in her account at any time. Complete the table and graph below to represent the situation.

x (years)	y (total)
0	
1	
2	
3	



8. Tyrone bought a new car for \$20,000. It depreciates at approximately 15% of its value each year. The equation that models the depreciated value is $y = 20{,}000(0.85)^x$. Complete the table and graph below.

x (age of car in years)	y (value of car)
0	
1	
2	
3	
4	



9. Jana’s grandfather put $1,000 in the bank for her when she was born. The account has been earning 2.5% compounded annually. Jana is now 18-years old and wants to take the money out so she can go to college. How much does she have in her account? Show how you got your solution.

10. You have just been notified that you are the only living descendent of a math-loving eccentric who put $10 in a bank account in 1755 and left it. Since then, the money has been collecting interest. The math-loving eccentric left instructions in his will that in the year 2010, if he had any living descendants who could determine the correct amount of money in the bank rounded to the hundredths place, that person could have the money. If not, the account was to be turned over to charity. Will you claim the money? Below is the information regarding the account. Show all your work clearly.

The $10 was deposited January 1, 1755, in a bank account that has an annual interest rate of 5%. Since then, the interest has been compounded annually. If you are correct in calculating the balance as of December 31, 2010, you can have the money.

Section 8.11 Population Growth

Practice Problems 8.11

For Problem 1-3, solve the word problem given.

1. In the population, health, and environment data and estimates for the countries and regions of the world, Germany has a -0.2% under the column titled, "Rate of Natural Increase." What does this mean?

2. If a country has a population of $1{,}543{,}200$ in 2015, and its growth rate is -0.2%, what is the equation for the total population by 2025? What will the population be in 2025?

3. Why might a country have no growth due to migration? If a country has 9 births per $1{,}000$ and 3 deaths per $1{,}000$ people, and no growth due to migration, what is the equation for population if the initial population is $1{,}002{,}000$ people?

For Problem 4-10, use table below that shows the four countries with the highest growth rates in 2015 to solve the problem given.

Country	Population	Percent of Growth	Percent of Growth as a Decimal Number
Lebanon	5,988,000	9.37%	
Zimbabwe	15,967,000	4.36%	
South Sudan	39,598,000	4.12%	
Jordan	6,459,000	3.86%	

4. List the percent of growth as a decimal number in the table for each of the four countries in 2015.

5. Write the population equations for the four countries in 2015.

6. What is the population of each of the four countries by 2025?

7. What is the population in 2025 for an island whose growth rate is 2.5% with an initial population of 25,000 in 2015?

8. A certain fungus covers the surface of the ground at a rate of four times its current surface every week. If the current fungus covers 2 square feet, how many square feet will the fungus cover in five weeks?

9. Whitetail deer are being reduced due to the construction of new housing. The population is presently 435 deer but is decreasing by 4% each year. What will the deer population be when the development is finished in five years?

10. Makenzie gets a job that pays $12.00 per hour the summer she graduates from high school. She works 40 hours a week for the twelve weeks of summer and deposits 60% of what she earns in a savings account that has an annual interest rate of 1.75% at the end of the summer. If she leaves the money in the savings account for four years while she is in college, how much money will she have in the account at the end of college?

Section 8.12 Comparing Power and Exponential Functions

Practice Problems 8.12

For Problem 1-4, use the tables in Problem 1 to solve the problem.

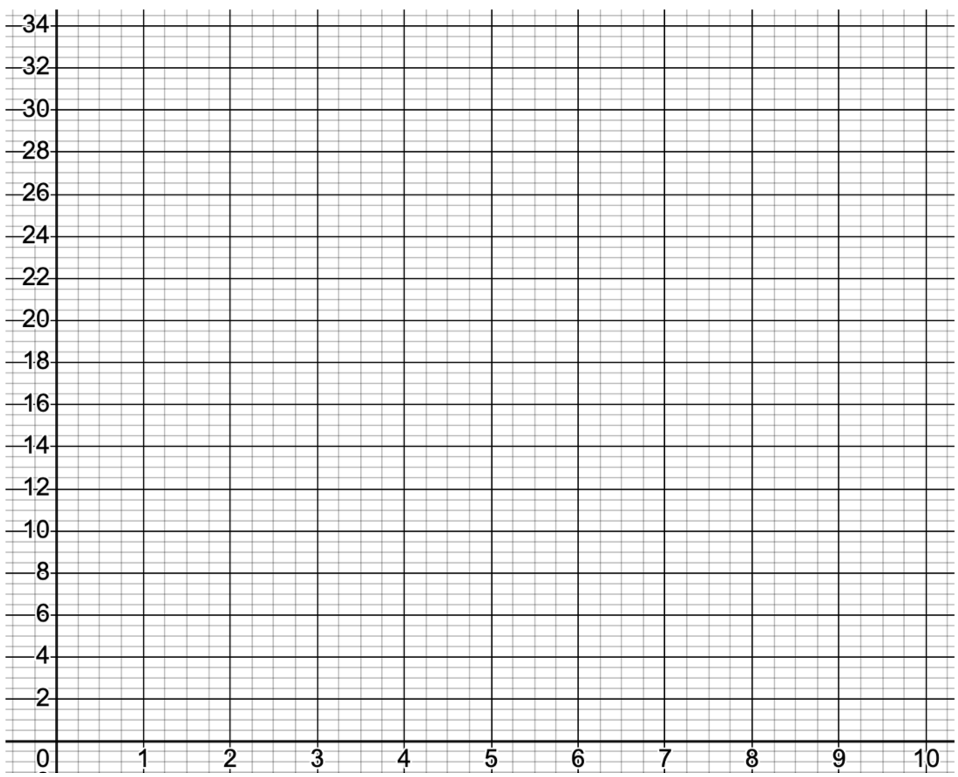
1. Complete the tables for $y = x^2$ and $y = 2^x$.

x	$y = x^2$
0	0
1	1
2	4
3	9
4	
5	
6	
7	

x	$y = 2^x$
0	1
1	2
2	4
3	8
4	
5	
6	
7	

2. Are there any values where the graphs intersect?

3. Draw the graphs in the Quadrant I using red and blue colored pencils.



4. What appears to be happening as x increases? Which graph seems to get steeper (increase more) in the long run?

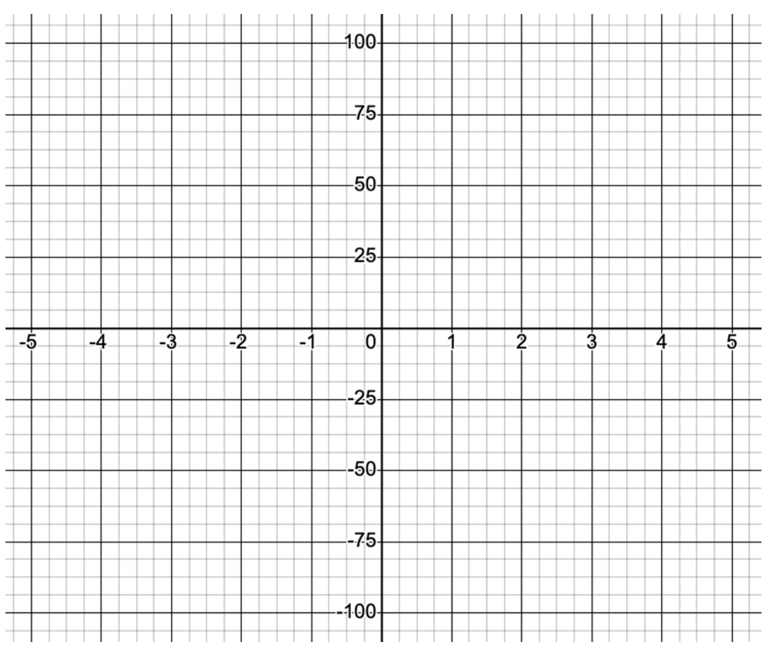
5. Fill in the blanks with $>$, $<$, or $=$: The value of $y =$ ____________ is greater for larger values of x from graphical observations in Problem 3. It could be said that 2^x ____________ x^2 in the long run.

For Problem 6-10, use the tables and graph in Problem 6 to solve the problem.

6. Compare $y = x^3$ and $y = 3^x$. Complete the tables and graph using red and blue colored pencils.

$y = x^3$	
x	y
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	

$y = 3^x$	
x	y
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	



7. In Problem 6…

a) For what integer values of x is $x^3 < 3^x$?

b) For what integer values of x is $x^3 = 3^x$?

c) For what integer values of x is $x^3 > 3^x$?

8. Based on your observations, does x^3 or 3^x appear to be greater in the long run?

9. Do you think it would be different if you used x^4 and 4^x rather than x^3 and 3^x?

10. Will b^x overtake x^4 for the values of $b = 2$ and $b = 3$ in the long run? Fill in the blanks below to help you answer the question.

	$\mathbf{2^x}$	$\boldsymbol{x^4}$	$\mathbf{3^x}$	$\boldsymbol{x^4}$
When $\boldsymbol{x = 10}$				
When $\boldsymbol{x = 20}$				

Section 8.13 Investigating Logarithms

Practice Problems 8.13

For Problem 1-10, convert the exponentials to logarithms and use the calculator to solve for x. Round your solution to the hundredth place.

1. $2^x = 11$
2. $5^x = 26$
3. $7^x = 48$
4. $3^x = 22$
5. $2^x = 100$
6. $10^x = 101$
7. $4^x = 23$
8. $5^x = 15$
9. $6^x = 18$
10. $6^x = 35$

Suppose you and a group of friends are shipwrecked on a tropical island. It is just beautiful and has coconuts, fish, bananas: everything you need to survive. The temperature is wonderful as well, a constant 80° almost every day. The cruise planner, Josephina, has a map of the island that allows for an escape route and is planning to present it at dinner.

At breakfast, one of the friends that does not want to leave the island slips Josephina a sleeping pill. It is time-delayed and temporarily cools the body using Newton's Law of Cooling just before Josephina wakes up. The sleeping body cools just like the cup of Joe from the Lesson Notes. While Josephina was sleeping, someone stole the map! Finding the culprit means retrieving the map and escaping the island!

At 1:00 PM, Jameson found Josephina sleeping. Jamison took Josephina's temperature and found it to be 96.1°. Her body was lying near the viewing spot for the volcano. Everyone took a picture while visiting the volcano viewing spot so there is a time stamp for when each was there.

At 2:00 PM, Josephina woke up. Jamison took her temperature, and it was 91.7°.

Below is when each person was at the volcano viewing spot:

Nancy: 12:13
Tammy: 12:15
Cayden: 12:18
Chrissy: 12:20
Teri: 12:22
Haley: 12:30
Debbie: 12:33
Cindy: 12: 34
Burgan: 12:36
Riley: 12:40
Katelyn: 12:42
Dusty: 12:45

Follow the steps below to find out who gave Josephina the sleeping pill and stole the map.
Find out when Josephina first fell asleep and the map was taken, then use the clues to find out who took it.

Step 1: Find T_E. Subtract the air temperature (T_A) from the temperature when Josephina was first found.

Step 2: Let the elapsed time be between the first temperature reading and the second temperature reading. Make sure the time is in minutes.
Use T_E, which you found in Step 1, and let T_B be the temperature of the body at 2:00 PM. Find the following values:

$t =$

$T_B =$

$T_A =$

$T_E =$

Step 3: Substitute these values into the equation when Josephina's body temperature was checked after one hour (the elapsed time is 60-minutes) and solve for b, the cooling rate, using roots, the inverses of the exponents.

$$T_B = T_E(b)^t + T_A$$

Step 4: Use the normal body temperature of 98.6° and substitute the value of b, the cooling rate from Step 3, into the equation and use logarithms to solve for time.

Step 5: Subtract the time from Step 4 from 1:00 PM when Josephina was first found sleeping to find out what time she fell asleep and who could have taken the map at that time.

Step 6: Using your equation from Step 5, let y_1 or $f(x_1)$ be the left side of the equation and let y_2 or $f(x_2)$ be the right side of the equation. Graph the two functions on the same coordinate grid on the calculator. Does this confirm the solution you found in Step 5 for the time? Let time be x. Find the point of intersection.
On your calculator, use "menu"- "analyze"- "graph"- "intersection" to find the exact point or go to the menu and trace the graph until you reach the point of intersection.
On Desmos® just click on the point. Use the wrench tool in the upper right-hand corner of the coordinate-grid to set the viewing window to $[-50, 50]$ for x and $[80, 100]$ for y.

Step 7: Let y_3 or $f(x_3)$ be 80°, the temperature on the island. Put it on the same coordinate grid as $f(x_1)$ and $f(x_2)$. Why does this become an asymptote for $f(x_1)$? The equation for T_B never goes below this line.

Who took the map?

Section 8.14 Module Review

For Problem 1-8, simplify the exponent given.

1. $2^5 + 1^8$

2. -3^4

3. $(-3)^4$

4. 4^{-3}

5. $(3y^2)^3$

6. $5^{\frac{1}{2}}$

7. $\frac{x^4}{6x^9}$

8. $\sqrt{\frac{x^2}{16}}$

For Problem 9 and 10, answer true or false for the exponential given.

9. $\frac{x^2y}{x} = \frac{y}{x}$

10. $\frac{x^5}{x^4} = \frac{6x^3}{6x^2}$

For Problem 11 and 12, find the explicit formula for each geometric sequence and write the next three terms in the sequence given.

11. $3, 15, 75, 375,$ ____________, ____________, ____________

12. $704, 352, 176,$ ____________, ____________, ____________

For Problem 13-16, answer the question given the exponential equation.

13. $y = 15(3)^3$

a) What is the initial term in the equation?

b) What is the common ratio of the equation?

14. $y = 3(2)^x$

a) What is the initial bee population if this equation represents the growth of a bee colony?

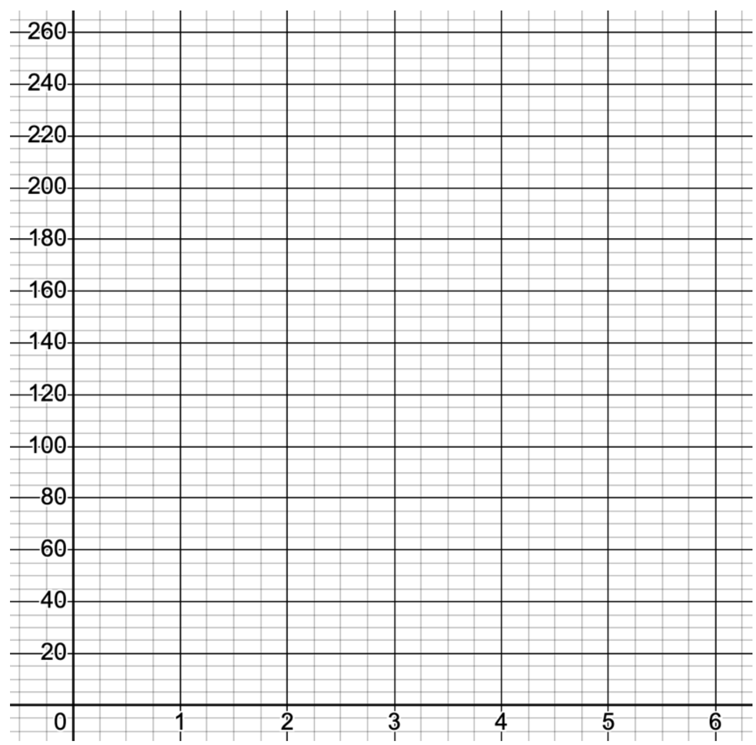
b) What is the rate of growth in terms of a percent?

15. If $y = 22(0.5)^x$ represents cell growth, is the population increasing or decreasing? How do you know?

16. In the equation $y = 22(0.5)^x$, what is the y-intercept of the graph of this equation? How do you know?

17. Make a table and graph for the equation $y = 4^x$. What is the y-intercept of the equation? How can you tell from the graph? How do you know from the table?

x	4^x
0	
1	
2	
3	
4	



18. Cole works for Craig and earns $430.00 to deposit in his savings account at 1.75% interest annually. How much will Cole have after five years?

19. If Alexandria deposits $320.00 in her savings account at the same interest rate as Cole in Problem 18, 1.75%, how many years will it take for her to save at least $450?

20. If Kyle deposits $725.00 in his account at an interest rate of 0.75% annually and Eli deposits $650.00 in his account at an interest rate of 1.5% annually, who will have more money saved for a car after four years of college?

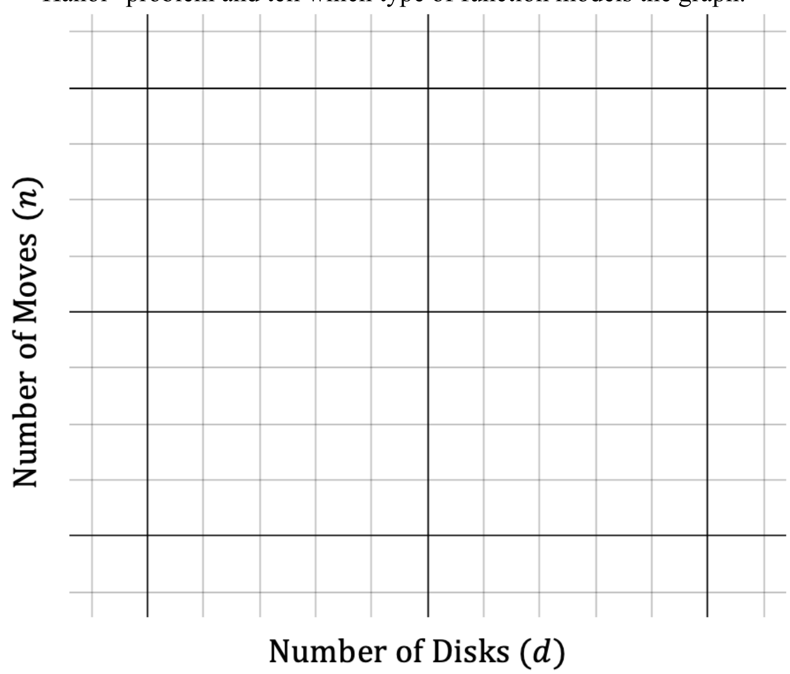
There is a problem of antiquity called “The Tower of Hanoi.” The tower consists of three pegs with eight graduated disks with holes in their centers. The graduated cylinders are arranged on one of the pegs with the largest on bottom and the smallest on top. The disks are moved to either of the pegs one at a time so that a larger disk never rests on a smaller disk. The goal is to move the disks to another peg in a minimum number of moves and determine the total minimum moves for any number of disks. Cut out the template of the tower base and the circular disks to answer the questions.

Complete the table for the number of moves to complete the task.

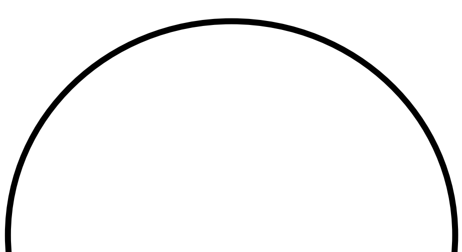
Number of Disks (d)	Number of Moves (n)
1	
2	
3	
4	
5	
6	

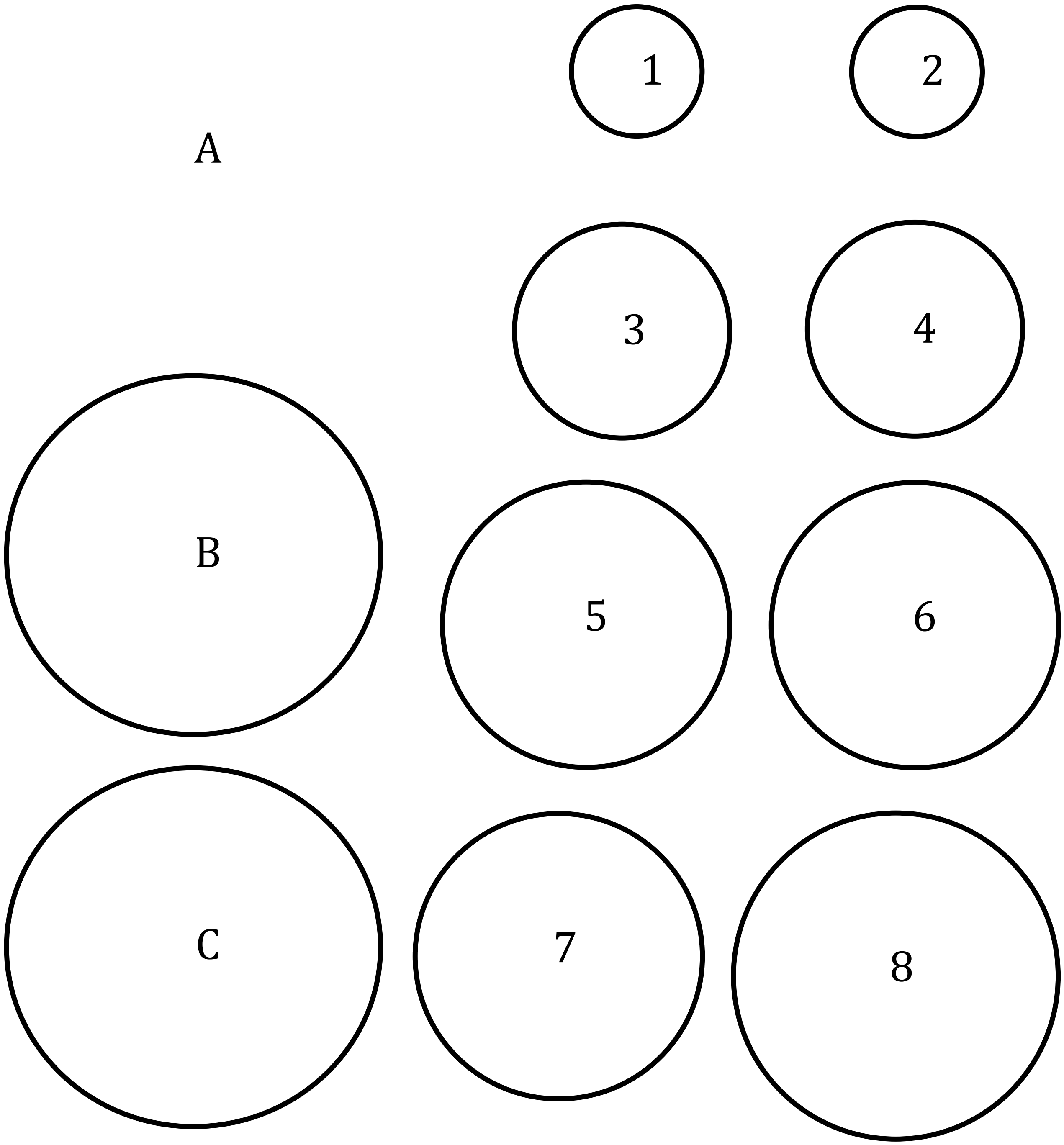
Use words to write the rule for what you do mathematically to the disks to get the number of moves.

Let d be the number of disks on the x-ais and let n be the number of moves on the y-axis. Graph the "Tower of Hanoi" problem and tell which type of function models the graph.



What is the equation for the total number of minimal moves given the number of disks?
(Hint: What is the base of the exponential function?)





What is the minimum number of moves to solve the problem if there are 64 disks?

<u>Section 8.15 Module Test</u>

For Problem 1-8, simplify the exponent given.

1. $2^2 + 3^5$

2. $(2^2)(3^3)$

3. $(-4)^{-2}$

4. $(7x)^2$

5. $(4y)(3y)^2$

6. $9^{\frac{1}{3}}$

7. $4^{\frac{2}{3}}$

8. $\sqrt[3]{\frac{27x^3}{125}}$

For Problem 9 and 10, answer true or false for the exponent given.

9. $\frac{x^7}{x^{10}} = x^{-3}$

10. $27^{\frac{1}{3}} - 16^{\frac{1}{4}} = 2$

For Problem 11 and 12, find the explicit formula for the geometric sequence and write the next three terms of the sequence given.

11. $2, 3, 4.5, 6.75,$ ____________, ____________, ____________

12. $7, 21, 63, 189,$ ____________, ____________, ____________

For Problem 13-16, answer the questions for the exponential equation given.

13. $y = 7.5(4)^x$

Given x represents days and y represents total grams of bacteria:

a) How many grams of bacteria will there be after one week?

b) How many days will it take for the bacteria to reach 120 grams?

14. In the equation from Problem 13, $y = 7.5(4)^x$:

a) What is the initial amount of bacteria in grams?

b) What is the growth factor?

c) What is the rate of growth in terms of a percent?

15. If $y = 31(\frac{1}{2})^x$ represents germ growth, is the germ population increasing or decreasing? How do you know?

16. In the equation $y = 31(\frac{1}{2})^x$, the y-intercept on the graph is 31. What is the value of x when $y = 31$? What does this mean?

For Problem 17-20, solve the word problem given.

17. Tony deposits $\$100$ in a bank at 1.68% interest. Write an equation for how much money Tony will have after n years. Let m represent the total amount of money.

18. Lance invested in a money market that is losing about 3% interest each year. Write an equation for how much money he would have after n years if he initially deposited $\$12{,}452.00$ in the money market. Let m be money and n be years.

19. At the end of the movie "God is Not Dead," the band Newsboys sing a song titled: "God is Not Dead." The band asks each member of the audience to text ten people saying: "God is Not Dead." If 5,836 people are in the

audience, how many people will receive the text saying, "God is Not Dead?" What if those who receive the text send it to ten more people and this happens again? How many people will receive the text after this happens three times? Write an exponential equation to model this situation.

20. How many total people, including those at the concert, will have heard the message: "God is Not Dead?"

Another fun game is called "Switch the Chips."
Below are the rules to play.

1. Put red or blue (or any two color you have) chips on a gameboard that is three squares long, five squares long, or seven squares long. Place the red chips on the left and the blue chips on the right so there is one space in the center that is an empty square.
2. Red chips may only move right, and blue chips may only move left.
3. There are only two possible moves you can make, a slide to an empty space or a jump over a chip but only in the direction given above.
4. Any color may move any number of times, but only one space at a time. The only other restriction is the direction: red right and blue left. Let the number of each color chip be n and the number of moves be $f(n)$. The game is over when all the red chips have been moved to the right side of the gameboard and all the blue chips have been moved to the left side of the gameboard.

On Gameboard 1, there is only one chip of each color.

R		B

On Gameboard 2, there are two chips of each color.

R	R		B	B

On Gameboard 3, there are three chips of each color.

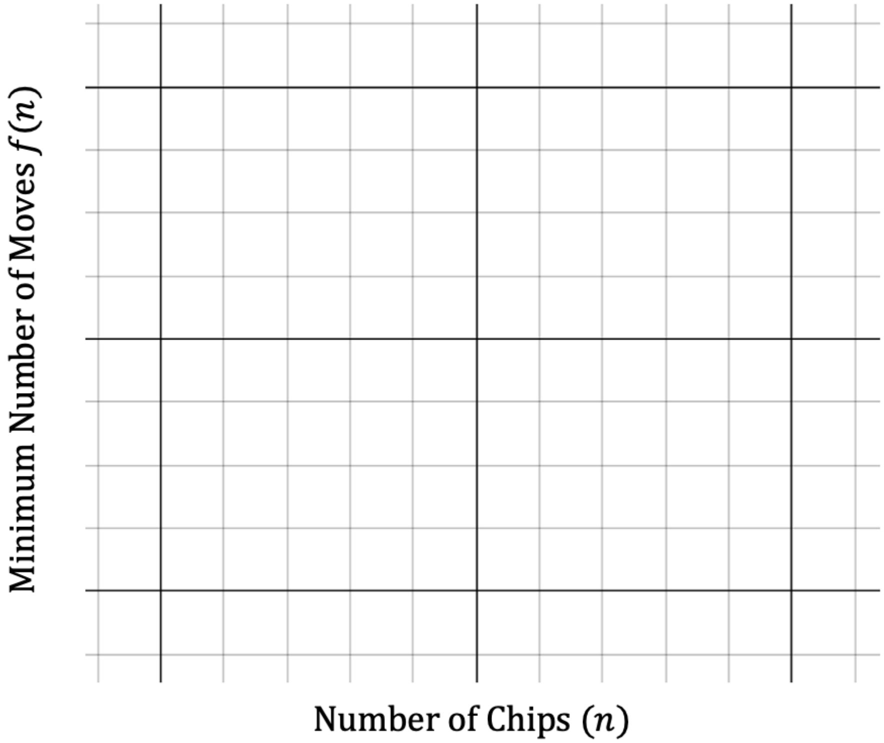
R	R	R		B	B	B

Use the attached Gameboards to play "Switch the Chips" and solve the problems given.

1. Complete the table for "Switch the Chips."

n	$f(n)$
1	
2	
3	
4	

2. Draw the graph for "Switch the Chips." Let n be the number of chips on the x-axis and let $f(n)$ be the minimum number of moves to switch the sides of the chips on the y-axis.



3. Does a quadratic or exponential function model the data?

4. What is the rule, in words, to describe what you do mathematically to n to get $f(n)$?

5. What is the equation for the minimum number of moves given the number of chips on one color? What is the minimum number of moves for 20 chips of each color?

Gameboard 1

Gameboard 2

Gameboard 3
