Module 7 Rational Expressions, Functions, and Equations

Section 7.1 Defining Rational Expressions

Practice Problems 7.1

For Problem 1-4, tell if the algebraic expression is rational. Explain why or why not.

1. $\frac{x}{x+3}$

2. $\frac{x^2-1}{\sqrt{x}}$

3. $\frac{x^2+3x-1}{x^{\frac{1}{2}}}$

4. $\frac{-4xy^2}{3xyz}$

5. Is $g(x) = \frac{x^2-1}{x}$ a rational function in its simplest form? Explain why or why not?

For Problem 6 – 9, simplify the rational algebraic expression.

6. $\frac{1-2x}{4x^2-1}$

7. $\frac{16-4y^2}{4y^2+2y-12}$

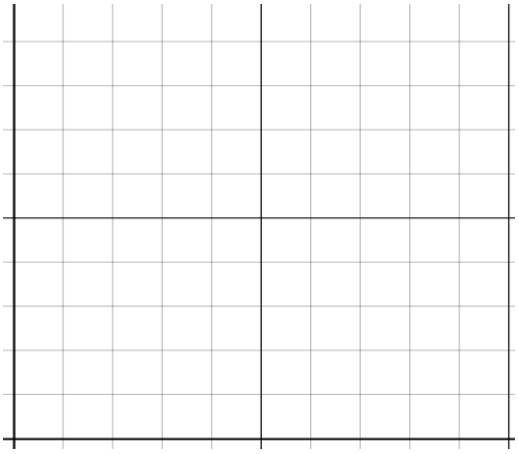
8. $\frac{15\,z^2-20z}{5z^3}$

9. $(y^2-9)(3+y)^{-1}$

10. To begin this experiment, you will need a piece of spaghetti. Tape it to the edge of a table so that 5 inches are hanging off the table. Hold it with a few pieces of tape the same space apart. Hang a cup from the spaghetti one inch from the end. The cup should be hanging from a string that is looped through both sides of the cup. Tape the string extending from the cup to the spaghetti so it doesn't slip as you add M&M's. Add the M&M's or Skittles into the cup one by one until the string breaks. Record the spaghetti length and the number of M&M's added to the cup until the breaking point.

Use another piece of spaghetti so that it is taped to the table and extends 4 inches beyond the table. Again, put the cup string one inch from the end at 3 inches and add M&M's to the cup until the spaghetti breaks. Repeat the experiment two more time to complete the table and graph.

Spaghetti Inches	Number of M&M's
5	
4	
3	
2	



What is the equation that models the data? Is it linear? Does it appear to be quadratic? Describe the line that is a good fit for the data.

This is a practical application of the type of experiment engineers would do to determine the amount of load that can be handled before the breaking point occurs, for example, an airplane wing.

Section 7.2 Graphing Rational Functions

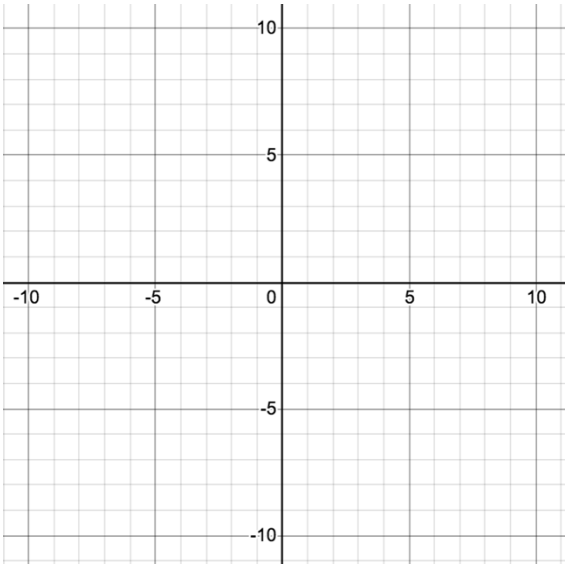
Practice Problems 7.2

For Problem 1-6, complete the table and graph, and then answer the following questions. Let $f(x) = \frac{1}{x}$.

a) What change was made from the parent function f to get the transformed function g?
b) What effect did this parameter change have on the graph?
c) What are the asymptotes?
d) At what values of x is the function undefined?

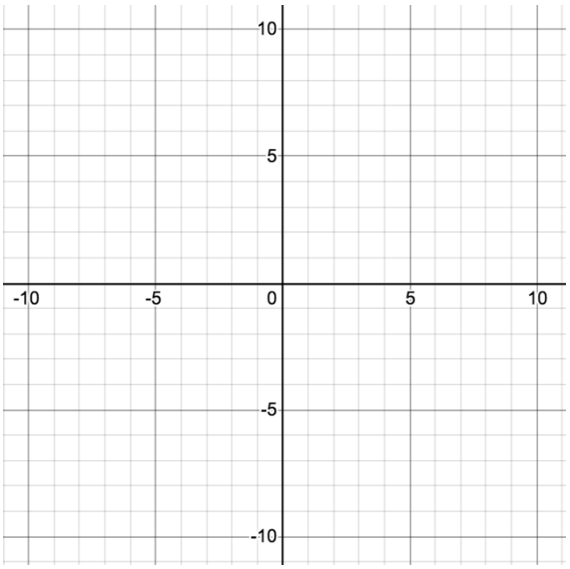
1. $g(x) = \frac{3}{x}$

x	$g(x)$
-3	
-2	
-1	
0	
1	
2	
3	



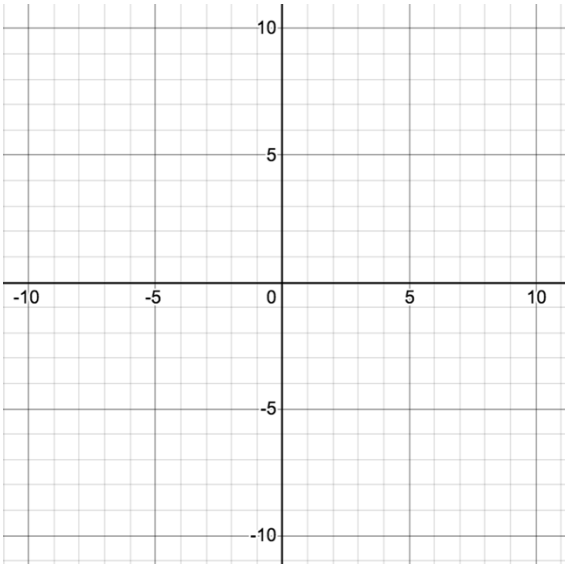
2. $g(x) = \frac{1}{x-4}$

x	$g(x)$
-3	
-2	
-1	
0	
1	
2	
3	
4	



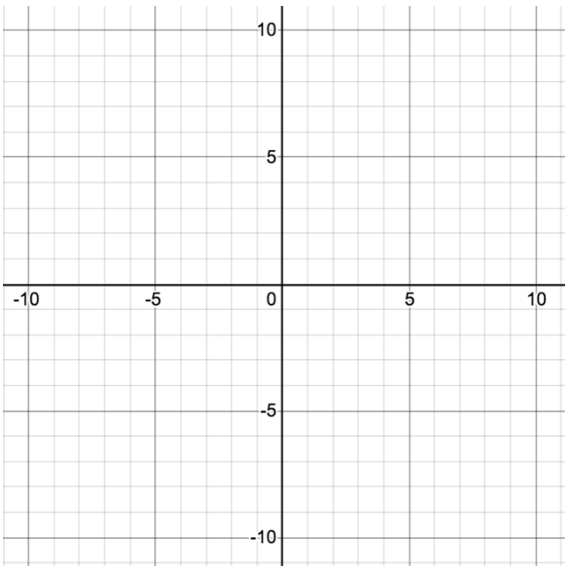
3. $g(x) = \frac{1}{x+2}$

x	$g(x)$
-4	
-3	
-2	
-1	
0	
1	
2	
3	



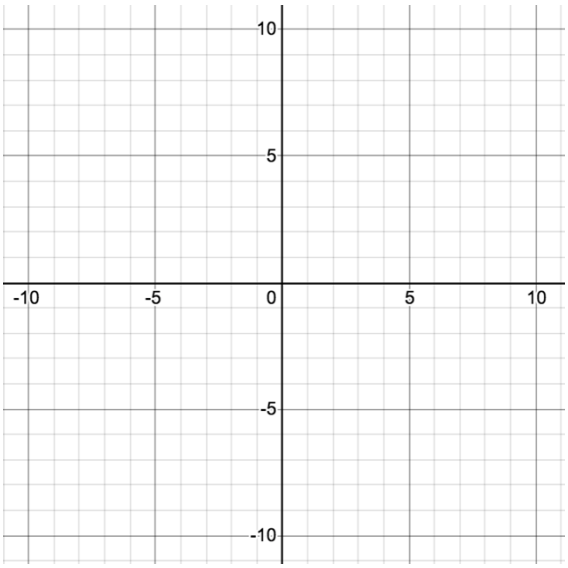
4. $g(x) = \frac{1}{x} + 5$

x	$g(x)$
-3	
-2	
-1	
0	
1	
2	
3	



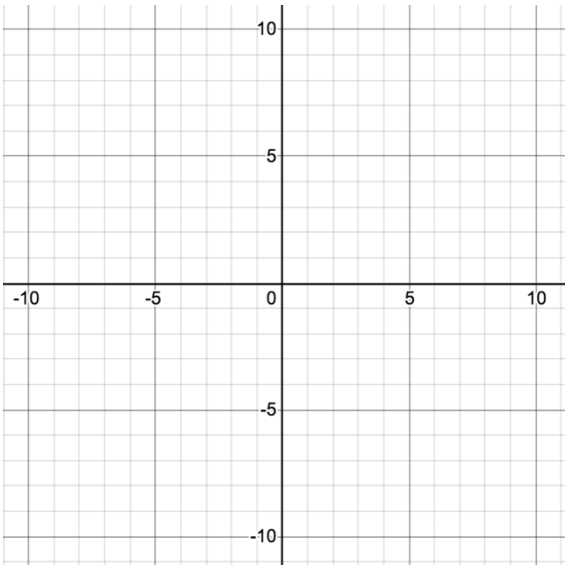
5. $g(x) = -\frac{1}{x}$

x	$g(x)$
-4	
-3	
-2	
-1	
0	
1	
2	
3	



6. $g(x) = \frac{1}{x} - 6$

x	$g(x)$
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	



For Problem 7 – 10, find the domain of each function and tell whether there is a removable or non-removable point of discontinuity.

7. $f(x) = \frac{1}{x^2-25}$

8. $f(x) = \frac{x+3}{x^2-9}$

9. $f(x) = \frac{1}{x^2-10x}$

10. $f(x) = \frac{x^3}{x^2+2}$

Section 7.3 Finding Equations of Rational Functions

Practice Problems 7.3

For Problem 1-6, write the equations that are transformations of the parent function $f(x) = \frac{1}{x}$.

1. Translate the graph horizontally 3 units right.

2. Translate the graph vertically 5.1 units up.

3. Translate the graph horizontally 2.3 units left.

4. Translate the graph vertically 1.8 units down.

5. Translate the graph horizontally 4 units to the right and vertically 6 units down.

6. Translate the graph horizontally 6 units left and vertically 3 units down.

For Problem 7-12, write the equations for the asymptotes of each function.

7. $f(x) = \frac{1}{x-3}$

8. $g(x) = \frac{1}{x+4.3}$

9. $f(x) = \frac{1}{x} - 1.7$

10. $g(x) = \frac{1}{x} + 5$

11. $h(x) = \frac{1}{x-5} + 1$

12. $k(x) = \frac{1}{x-1} - 7$

For Problem 13-16, write the equations for the rational functions with the following asymptotes that are translations of the function $f(x) = \frac{1}{x}$.

13. Vertical asymptote: $x = -3$

    Horizontal asymptote: $y = -1$

14. Vertical asymptote: $x = 9$

    Horizontal asymptote: $y = 2$

15. Vertical asymptote: $x = -3$

    Horizontal asymptote: $y = -5$

16. Vertical asymptote: $x = 0$

    Horizontal asymptote: $y = 8$

For Problem 17-20, write the steps of the transformations to complete the graph from the given parent function $f(x) = \frac{1}{x}$.

17. $g(x) = \frac{2}{x+3}$

18. $h(x) = \frac{-2}{x} + 5$

19. $j(x) = \frac{-3}{x-4}$

20. $k(x) = \frac{-4}{x+1} - 7$

Section 7.4 Transformations of Rational Functions

Practice Problems 7.4

For Problem 1-5, name the domain of the function.

1. $f(x) = \frac{3x}{x+6}$

2. $g(x) = \frac{x^2}{x-2}$

3. $h(x) = \frac{1+x}{1-x}$

4. $f(x) = \frac{x-5}{x+3}$

5. $g(x) = \frac{x}{x+10}$

For Problem 6-10, tell how many vertical and horizontal asymptotes each function has.

6. $f(x) = \frac{1}{x-2}$

7. $g(x) = \frac{2x^3+15}{x^3}$

8. $h(x) = \frac{3x^2+5}{x+1}$

9. $f(x) = \frac{5x}{x^2+1}$

10. $g(x) = \frac{4x^3}{x^2+2}$

For Problem 11-15, name the vertical and horizontal asymptotes of each function.

11. $f(x) = \frac{2}{x^2-4}$

12. $g(x) = \frac{2x^2+4}{3x^2-12}$

13. $h(x) = \frac{4x^3+40}{2x^3}$

14. $f(x) = \frac{1}{x+2}$

15. $h(x) = \frac{11x^3+x}{3x+3}$

For Problem 16-20, find the two vertical asymptotes of each function.

16. $f(x) = \frac{1}{x^2-9}$

17. $g(x) = \frac{5}{x^2-25}$

18. $h(x) = \frac{-2}{x^2+3x}$

19. $f(x) = \frac{x}{4x^2-81}$

20. $g(x) = \frac{x^3}{x^2-2x-8}$

Section 7.5 Direct Variation

Practice Problems 7.5

1. Various groups experimented with sucrose solutions of 1%, 5%, 10%, 15%, and 20%. The table below gives the results for density at room temperature. Plot the percentage of sugar on the x-axis and plot known density on the y-axis. What is the line of best fit?

Percentage of Sugar (x)	1%	5%	10%	15%	20%
Density at Room Temperature (y)	1.003	1.02	1.03	1.059	1.07

2. Using the graph from Problem 1, and the density of diet Cola being 0.972963 g/mL, estimate the percentage of sugar of the beverage.

3. Students calculated the percentage of sugar in a regular Cola to be 12.2%. They found the experimental percentage of sugar from the same beverage to be 11%. Calculate the percentage of error using the formula below:

$$Percent\ Error = \frac{|Calculated\ Value - Experimental\ Value|}{Calculated\ Value} \cdot 100\%$$

4. Below is the data for the standard solution of 20% sucrose. Complete the table. The mass of the graduated cylinder is 94.72 g.

Volume of 20% Sucrose Solution (mL)	Mass of Graduated Cylinder and Solution (g.)	Mass of Solution (g.)
10.0	105.98	
20.0	116.83	
30.0	127.37	
40.0	138.25	
50.0	149.04	
60.0	159.46	
70.0	170.62	
80.0	181.28	

5. What is the slope of the 20% sucrose standard solution in Problem 4 and what does it represent?

6. What is the line of best fit for Problem 4?

7. The table below is the data for Sweet Tea. Complete the table and find the slope and line of best fit. What is the density of Sweet Tea? The mass of the graduated cylinder is 95.58 g.

Volume of Sweet Tea (mL)	Mass of the Graduated Cylinder with Sweet Tea (g.)	Mass of the Sweet Tea (g.)
10.0	106.13	
20.0	116.35	
30.0	126.88	
40.0	137.10	
50.0	147.22	
60.0	157.43	
70.0	168.47	
80.0	178.44	

8. What is the percentage of sugar for sweet tea if the nutrition label says there is 40 g. of sugar in 350 mL of beverage?

Below is a table of experimental results for three beverages. Use the data for Problem 9 and 10.

Beverage	**Density**	**Percentage of Sugar (Experimental)**	**Nutrition Label**
Grape Juice	1.05 g/mL	13.0	40 g/240 mL
Apple Juice	1.04 g/mL	16.0	30 g/355 mL
Orange Juice	1.03 g/mL	15.2	25 g/240 mL

9. Calculate the percentage of sugar for each beverage.

10. Find the percentage of error for each beverage.

Section 7.6 Correlation Coefficient

Practice Problems 7.6

For homework today, you will be given data from another Cola experiment. The data for x (volume of the Standard Solution) and y, actual value (mass of Cola beverage), is completed for you in the table. Follow the steps of the worksheet to complete the table and fill in the blanks.

After completing the table and filling in the blanks, answer the following question:

What reasonable assumptions can be made about the density and sugar content of a beverage?

1. Complete Lists and Spreadsheets from the table below with the values for x and y from the problem.
2. Find the linear regression equation ($y = mx + b$) and let that be $\hat{y}$. This is the predicted value of y based on the equation. Complete the table below.
3. Find the difference between the predicted value, $\hat{y}$, and the actual value of y from the experiment. This is called the residual. Complete the table.
4. Find the squares of the residuals and complete the table.
5. Add up all the squares of the residuals. This is called the sum of the squares of the residuals, or SS_{res}; in this experiment, $SS_{res} =$ _____.
6. Add up all the values of the y coordinates and divide by the total number of y values. This is called the mean, or $\bar{y}$. The mean is _____.
7. Find the difference between the y values and the mean. This is called the deviation. Complete the table below.
8. Find the squares of the deviation and complete the table below.
9. Add up all the squares of the deviations. This is called the sum of the squares of the deviations or $SS_{dev} =$ _____.
10. The SS_{res} is a minimum of all possible values. The deviation is from the horizontal (constant function). The residual is from the line of regression and is smaller. If you remove the fractional difference, or SS_{dev}, of one from the other you get an even better fit. This is called the coefficient of determination.

$$r^2 = \frac{SS_{dev} - SS_{res}}{SS_{dev}} \quad r^2 = \_\_\_\_\_$$

11. The r is called the correlation coefficient and it measures the fit of the data to the function. If it is 1 or -1, it is a perfect fit. The correlation between the function and data can be strong or weak, or not at all if $r = 0$.

$$\sqrt{r^2} = \_\_\_\_\_$$

12. The function that identifies a pattern in Chemistry that is a theoretical law would give an exact fit, but when you collect data from an experiment, it is not an exact fit. Is your correlation strong or weak? Is your correlation positive or negative? Explain why based on what you have learned through the beverage density experiments.

Volume	**Actual Value**	**Predicted Value**	**Residual**	**Residual Squared**	**Deviation**	**Deviation Squared**
x	y	$\hat{y}$	$y - \hat{y}$	$(y - \hat{y})^2$	$y - \bar{y}$	$(y - \bar{y})^2$
10.0	10.21					
20.0	20.34					
30.0	30.95					
40.0	46.63					
50.0	51.09					
60.0	61.39					
70.0	72.11					

Now answer the question: What reasonable assumptions can be made about the density and sugar content of a beverage?

Section 7.7 Inverse Variation

Practice Problems 7.7

For Problem 1-10, solve the word problem.

1. The resistance of a wire varies inversely with its cross-sectional area, A. The radius of the wire is 2 mm. and the resistance of the wire is 0.025 Ω. What is the constant of variation, *k*?

2. Find the resistance of the same length of the same type of wire in Problem 1, but the wire has a radius of 0.05 mm.

3. The intensity of sound, I (measured in watts per square meters), varies inversely with the square of the distance to the emission of the sound. The intensity of noise from a jet engine is 0.1 watts per square meter from 10 meters away from the engine. What is the constant of variation?

4. A flight attendant boards a plane 200 meters away from a plane that has landed but still has the engine running. What is the intensity of sound of the jet engine?

5. Find the constant of variation for the weight of a body, *w*, that varies inversely as the square of its distance, *d*, from the center of the earth, which is approximately 3,959 miles. The body weighs 140 pounds.

6. Using the weight of the body of 140 pounds in Problem 5, how much would the body weigh if it were 500 miles above the earth's surface?

7. The formula for power is $P = I \cdot V$ (measured in watts). What is the power of the 8.3 amps blow dryer with 120-volts?

8. What would be the power of the same blow dryer plugged into a 240-volt line in Europe (assume a converter is used)?

9. In Ohm's Law, a circuit with a potential of 1 volt has a current of 1 amp and a resistance of 1 ohm so that $1V = 1A \cdot 1\Omega$. If the resistance is increased to 2Ω and the voltage is the same, what is the current?

10. If the resistance in Problem 9 is decreased to 0.05Ω and the voltage stays the same, what is the current?

For Problem 11-20, solve the word problem.

11. You pay an annual fee of $120 to belong to a movie club. Each time you watch a movie on television you pay $3 on top of the annual fee. Write an equation to model the annual cost per movie including the annual fee of $120.

12. Given the costs in Problem 11, if you watch 55 movies during the year, what is the average cost of each movie?

13. Given the costs in Problem 11, when would you pay less than $4 per movie?

14. Given the costs in Problem 11, how much is each movie with the fee if you watch 120 movies?

15. If the degree of the numerator of a rational function is exactly one degree more than the degree of the denominator, there is a slanted asymptote on the graph. If $f(x) = \frac{x^2+1}{x-1}$, there will be a slanted asymptote. Divide $x^2 + 1$ by $x - 1$. Write the remainder as a quotient.

16. In Problem 15, the part of the quotient that is not the remainder is the slanted asymptote. What is the equation of the line that is the slanted asymptote?

17. What is the vertical asymptote of the function $f(x) = \frac{x^2+1}{x-1}$?

18. How do you know there is no horizontal asymptote in Problem 17?

19. For the equation in Problem 17, sketch the vertical and horizontal asymptotes as well as the graph and the curves in between the upper boundaries and lower boundaries.

20. Why doesn't $f(x) = \frac{x^2-2x-3}{x+1}$ have a slanted asymptote?

Section 7.8 Simplifying Rational Expressions

Practice Problems 7.8

For Problem 1-10, tell whether the equation is true or false.

1. $\frac{x}{x+1} = \frac{x^2}{x^2+1}$

2. $\frac{x}{x^2+x} = \frac{x}{x(x+1)}$

3. $\frac{x}{x^2+x} = \frac{1}{x+1}$

4. $\frac{(x+3)(x-1)}{x+2} = \frac{x^2-3x+3}{x+2}$

5. $\frac{3x^2-5x+1}{3x^2-5x-1} = 1$

6. $\frac{2x^2-4}{2(x^2-2)} = 1$

7. $\frac{x^2+4x}{2x+8} = \frac{x}{2}$

8. $\frac{x^2+4x}{2x+8} = \frac{1}{2}x$

9. $\frac{x^2-4}{x^2-1} = \frac{(x+2)(x-2)}{(x+1)(x-1)}$

10. $\frac{x^2+2x-3}{x^2-1} = \frac{x+3}{x+1}$

For Problem 11-13, tell whether or not the expression is simplified.

11. $\frac{x-7}{9(x+1)}$

12. $\frac{(x+4)(x-2)}{6x(x+4)}$

13. $\frac{(x+1)^2}{(x+1)(x-4)}$

For Problem 14-16, tell what the domain of each expression is and find the real zeroes of the denominator.

14. $\frac{x+1}{x-3}$

15. $\frac{x+5}{x}$

16. $\frac{x-2}{3x-3}$

For Problem 17-20, simplify each expression and tell which values of x make the expression true.

17. $\frac{(x+3)}{(x+3)^2x}$

18. $\frac{2x+8}{x^2-16}$

19. $\frac{2x^2-2}{(x+1)^2}$

20. $\frac{10-5x}{(x-2)(x+3)}$

Section 7.9 Adding Rational Expressions

Practice Problems 7.9

For Problem 1-10, find the least common denominator of the rational expressions.

1. $\frac{3x}{x}, \frac{4}{x-1}$

2. $\frac{x}{x+1}, \frac{2}{x}$

3. $\frac{x+3}{x-2}, \frac{x-4}{x+2}$

4. $\frac{3x+1}{3x-2}, \frac{x}{x+1}$

5. $\frac{x-5}{x+5}, \frac{x+5}{x-5}$

6. $\frac{3}{y}, \frac{1}{y^2}$

7. $\frac{-5}{y+2}, \frac{y}{y^3}$

8. $\frac{3}{y-1}, \frac{y+1}{y}$

9. $\frac{1}{y^2+3y+2}, \frac{-6y}{y+1}$

10. $\frac{5}{y}, \frac{7x}{y^2}, \frac{x+y}{y^5}$

For Problem 11-20, add the rational expressions.

11. $\frac{2y^2-2}{(y-5)(3y+1)} + \frac{y}{3y+1}$

12. $\frac{8}{5y} + \frac{5}{4y}$

13. $\frac{9-3y}{(y+3)(y-3)}+\frac{2y}{y+3}$

14. $\frac{y^2}{y^2-y-30}+\frac{3y+5}{y+5}$

15. $\frac{y+3}{y^2-3y-28}+\frac{2y-1}{3y^2+13y+4}$

16. $-\frac{4}{x}+\frac{2}{3x}$

17. $\frac{2}{x-3}+\frac{1}{x}+\frac{2x}{x+2}$

18. $\frac{4x-1}{x-2}+\frac{3}{x^2-4}$

19. $\frac{7}{2(x-5)}+\frac{x+2}{3x}$

20. $\frac{1}{x^2-3x-4}+\frac{1}{x-1}$

Section 7.10 Subtracting Rational Expressions

Practice Problems 7.10

For Problem 1-10, find the least common denominator of the rational expressions.

1. $\frac{5x}{x^2}, \frac{10}{-x}$

2. $\frac{6}{x-4}, \frac{2x}{x}$

3. $\frac{-7}{x-2}, \frac{8x}{x+7}$

4. $\frac{3x-5}{2x-4}, \frac{x^2+1}{x-2}$

5. $\frac{-2x+1}{4x}, \frac{x}{8x}$

6. $\frac{y}{2}, -\frac{2}{y}$

7. $\frac{y+1}{y-2}, \frac{y}{y^3}$

8. $\frac{8}{x-2}, \frac{9}{x+2}$

9. $\frac{4}{y-1}, \frac{y-1}{8}$

10. $\frac{2y}{y^2-3y}, \frac{4-y}{y}$

For Problem 11-20, subtract the rational expressions.

11. $\frac{3y+1}{y^2-4} - \frac{3}{y-2}$

12. $\frac{5}{y} - \frac{10}{y^4}$

13. $\frac{2y}{y+1} - \frac{6}{y-1}$

14. $\frac{y}{3} - \frac{1}{6+y}$

15. $\frac{2}{y+2} - \frac{5}{y-2} - \frac{y}{3}$

16. $\frac{2-4x}{x^2-x-6} - \frac{4}{x-3}$

17. $\frac{10x}{2x^2-4} - \frac{4}{x-1}$

18. $\frac{9}{3(x+5)} - \frac{x+1}{3x}$

19. $\frac{x^2}{3x+5} - \frac{7}{x+7}$

20. $\frac{4x+1}{x^2-1} - \frac{3x}{x+1}$

Section 7.11 Multiplying Rational Expressions

Practice Problems 7.11

For Problem 1-5, simplify the rational expression.

1. $\frac{(2y-1)(y-4)^2}{(y-4)(2y-1)}$

2. $\frac{(y+1)(y^2+y-3)}{(y^2-1)(y+2)}$

3. $\frac{7y^2-22y+3}{3y^2-7y-6}$

4. $\frac{(y+6)^5(y-5)^4}{(y-5)^3(y+6)^7}$

5. $\frac{x(x+1)}{5x^2(x-2)}$

For Problem 6-10, tell whether the phrase is true or false given the rational expression below:

$$\frac{x^2-6x-7}{x^2-2x-3}$$

6. The rational expression is simplified.

7. The real zeroes of the denominator are $x=-1$ and $x=3$.

8. The rational expression is cannot be evaluated when $x=4$.

9. In its' simplest form, the expression is $\frac{x-7}{x-3}$.

10. The expression $\frac{2(x-7)}{4(x-3)}$ is an equivalent rational expression.

For Problem 11-20, multiply the rational expression and simplify the answers.

11. $\frac{2y^2+3y-35}{y^2+3y-10} \cdot \frac{4y-12}{2y^2-13y+21}$

12. $\frac{y-1}{y-4} \cdot \frac{y-4}{y-1}$

13. $\frac{y}{x} \cdot \frac{x^2+4}{y-1}$

14. $\frac{(y+6)(y-3)}{(y-3)(2y+5)} \cdot \frac{(y-3)(2y+5)}{(y+1)(y+6)}$

15. $\frac{y^2-16}{(y-2)(y+1)} \cdot \frac{y^3}{(y+4)(y-4)}$

16. $\frac{x}{3(x+1)^2} \cdot \frac{9(x+1)}{x^5}$

17. $\frac{(x+3)(x-2)}{(x+4)^5} \cdot \frac{(x+4)^4}{(x-3)(x+2)}$

18. $\frac{x^2+8x-9}{3x^2+10x-8} \cdot \frac{2x^2+5x-12}{2x^2-5x+3}$

19. $\frac{x}{x+1} \cdot \frac{3}{x^5} \cdot \frac{x+1}{12}$

20. $\frac{11(3x+1)}{(2x+3)} \cdot \frac{(2x+3)}{22(4x-9)}$

Section 7.12 Dividing Rational Expressions

Practice Problems 7.12

For Problem 1-10, divide the rational expressions.

1. $\frac{2x-1}{x^2-x-2} \div \frac{x+2}{x+1}$

2. $\frac{y^2-4y-5}{y^2+4y-12} \div \frac{y^2-2y-3}{y^2-4y+4}$

3. $\frac{x^2+20}{5x} \div \frac{6+5x}{5x}$

4. $\frac{5}{12y} \div \frac{4y}{7}$

5. $\frac{y^2-9}{y+5} \div \frac{y^2+6y+9}{y^2+3y-10}$

6. $\frac{x+1}{x^2-x-2} \div \frac{(x-1)(x-3)}{x^2-9}$

7. $\frac{y^2+5y-6}{y+1} \div \frac{y^2+7y+6}{2y^2-2y}$

8. $\frac{10}{y^2} \div \frac{3}{y(y+1)}$

9. $\frac{x^2-9}{x^2-4x-5} \div \frac{x^2-8x+15}{x+3}$

10. $\frac{x}{x+2} \div \frac{x+7}{x^2-4}$

For Problem 11-15, divide the complex fractions by making one fraction in the numerator and another in the denominator, and then multiply by the reciprocal of the fraction in the denominator.

11. $\frac{\frac{x}{3}+\frac{4}{x}}{\frac{2}{7x}+1}$

12. $\frac{1+\frac{9}{x}}{1-\frac{6}{x}}$

13. $\frac{\frac{8}{x}+2}{\frac{1}{4x}-8}$

14. $\frac{4-\frac{1}{x^2}}{\frac{1}{4x^2}-2}$

15. $\frac{\frac{x}{x-1}-\frac{1}{3}}{\frac{10}{x}+\frac{x^2}{x-1}}$

For Problem 16-18, divide the fractions by finding a Least Common Denominator of the fraction in the numerator and the denominator and then simplifying the fraction.

16. $\dfrac{1-\frac{x}{x+4}}{\frac{x+3}{x^2-16}}$

17. $\dfrac{\frac{1}{x-2}+\frac{1}{x+2}}{\frac{x}{x-2}-\frac{x}{x+2}}$

18. $\dfrac{\frac{x+5}{x^2-3x-10}}{\frac{x^2-25}{x^2-x-6}}$

For Problem 19 and 20, use either method to simplify the rational equation and then solve.

19. $\dfrac{\frac{x-9}{5}}{\frac{1}{2}+\frac{x+4}{10}} = -2$

20. $\dfrac{\frac{3}{x-1}}{\frac{2}{x}} = 1$

Section 7.13 Solving Rational Equations

Practice Problems 7.13

For Problem 1-10, solve the rational equation. Check for any extraneous solutions.

1. $\frac{5x-2}{x-1} = \frac{2x-3}{2}$

2. $\frac{2x+1}{x+3} = \frac{x+6}{x}$

3. $\frac{4}{6x+12} = \frac{2x+4}{2x^2-8}$

4. $\frac{2x^2-4x-6}{3(x-2)} = \frac{x-3}{3}$

5. $\frac{x+5}{x-1} - 3 = \frac{x+6}{x}$

6. $\frac{x+3}{x} + 1 = \frac{1}{x-1}$

7. $\frac{2x+4}{x} = \frac{2x-2}{x-2}$

8. $\frac{x}{x-1} = \frac{4}{x+3}$

9. $\frac{4x+20}{2x+22} = \frac{2x}{2x-2}$

10. $\frac{9}{2x-3} + \frac{x}{x+2} = 2$

For Problem 11-16, fill in the blanks.

11. A rational expression has a numerator and denominator that are both _____.

12. The real _____ of a denominator is/are not part of the domain of the rational expression.

13. To add or subtract rational expressions you must first find the _____ _____ _____.

14. A rational expression, which has a fraction in both the numerator and denominator, is called a _____ fraction.

15. Repaying a loan or an installment plan is called _____.

16. The amount of money received for a loan is called the _____.

For Problem 17-20, solve the word problem.

17. Find the monthly payment on a house loan of $120,000.00 if it is to be paid in monthly installments over 15 years at 4.5% interest.

18. What would the monthly payment be on the same house loan from Problem 17 if the term were 30 years instead of 15 years?

19. A container of 100 milliliters of lemonade is made up of 70% water and 30% pure lemon juice. How many milliliters of pure lemon juice needs to be added to the lemonade so that the concentration is 60% pure lemon juice?

20. When milk is labeled 2% that means that 2% of the gallon of milk is fat (this is pure fat). A gallon jug, or 3,780 milliliters, of Vitamin D milk is 3.5% fat. How many milliliters of half and half (50% pure fat) must be added to a 2% gallon of milk to get 2 gallons of milk with the same fat content as vitamin D milk?

Section 7.14 Module Review

1. Circle the expressions that are not rational expressions:

a) $\frac{x^{-2}}{y^{-4}}$ b) $x^{\frac{1}{5}} + y^{\frac{1}{3}}$

c) $-\frac{5x}{7y}$ d) $\frac{x+y}{x-y}$

2. Make a table and graph for the following function:

$$f(x) = \frac{1}{x} - 3$$

For Problem 3 and 4, write the equation for the transformations of the parent function $f(x) = \frac{1}{x}$.

3. 5 units right

4. 1 unit right and 4 units up

For Problem 5 and 6, write the equations for the asymptotes given the following hyperbolas:

5. $f(x) = \frac{1}{x+7}$

6. $f(x) = \frac{1}{x-2} - 2$

For Problem 7 and 8, write the equation for the rational function with the following asymptotes that are transformations of the function $f(x) = \frac{1}{x}$.

7. Vertical Asymptote: $x = 8$

   Horizontal Asymptote: $y = \frac{1}{3}$

8. Vertical Asymptote: $x = 0$

   Horizontal Asymptote: $y = -5$

For the Problem 9 and 10, given the function $f(x) = \frac{p(x)}{q(x)}$, name the vertical and horizontal asymptote and sketch a graph of the function.

9. $f(x) = \frac{x-1}{x+1}$

10. $f(x) = \frac{1}{x^2+4}$

For Problem 11-13, name the domain of the function.

11. $f(x) = \frac{2}{x+4}$

12. $g(x) = \frac{x}{x-3}$

13. $h(x) = \frac{x^2}{x}$

For Problem 14 and 15, simplify the rational function.

14. $f(x) = \frac{x^2-2x-3}{(x+5)(x-3)}$

15. $g(x) = \frac{x+3}{x^2+6x+9}$

For Problem 16-19, perform the operations on the rational expression.

16. $\frac{2x}{x-2} + \frac{5}{x+6}$

17. $\frac{1}{x+3} - \frac{3x-x^2}{x^2-9}$

18. $\frac{x^2+3x}{4x^3+12x^2} \cdot \frac{x^2+x-2}{x^2+6x+8}$

19. $\frac{x^2-4x-12}{x^2-6x} \div \frac{6}{3x-24}$

For Problem 20, solve the word problem.

20. Find the monthly payment for a $20,000.00 small business loan at a rate of 9.5% for 10 years.

Section 7.15 Module Test

1. Circle the expressions below that are not rational expressions:

a) $\frac{2x}{y-4}$ b) y^{-5}

c) $(x+y)^{\frac{1}{5}}$ d) $\frac{x}{y}$

2. Make a table and graph for the following function:

$$f(x) = \frac{1}{x+2}$$

For Problem 3 and 4, write the equation for the transformation of the parent function $f(x) = \frac{1}{x}$.

3. 3 units up

4. 2 units left and 1 unit down

For Problem 5 and 6, write the equations for the asymptotes for the following hyperbolas:

5. $f(x) = \frac{1}{x-2.3}$

6. $f(x) = \frac{1}{x+4} + 1$

For Problem 7 and 8, write the equation for the rational function with the following asymptotes that are transformations of the function $f(x) = \frac{1}{x}$.

7. Vertical asymptote: $x = -2$
   Horizontal asymptote: $y = 2$

8. Vertical asymptote: $x = -4$
   Horizontal asymptote: $y = 0$

For Problem 9 and 10, given the function $f(x) = \frac{p(x)}{q(x)}$, name the vertical and horizontal asymptotes and sketch the graph of the function.

9. $f(x) = \frac{2x^2}{x^2-9}$

10. $f(x) = \frac{1}{x^2-2}$

For Problem 11-13, name the domain of the function.

11. $f(x) = \frac{x+3}{x-5}$

12. $g(x) = \frac{2x-8}{x^2}$

13. $h(x) = \frac{-1}{x^2-4}$

For Problem 14 and 15, simplify the rational function.

14. $f(x) = \frac{3x(x-4)}{(x-4)(x+4)}$

15. $g(x) = \frac{(x-4)(x-3)}{(x-3)(x+1)}$

For Problem 16-19, perform the operations on the rational expression.

16. $\frac{x}{x+3} + \frac{x-4}{x}$

17. $\frac{3x-4}{x^2+3x-10} - \frac{5}{x-2}$

18. $\frac{2x+3}{4x^2-9} \cdot \frac{6x^3-9x^2}{x^3+10x^2}$

19. $\frac{x+4}{x-3} \div \frac{x^2+3x-4}{x^2-4x+3}$

20. You take out a used car loan for $6,000 with an interest rate of 8%. You want to pay it off in the least amount of time possible but can only afford $150 a month. Which of the terms below should you accept?

a) 12 months

b) 24 months

c) 36 months

d) 48 months