Geometry and Trigonometry Module 6 Triangles

Section 6.1 Introduction to Triangles

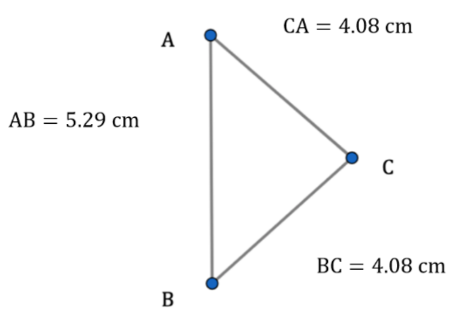
Practice Problems 6.1

For Problem 1-8, use the given information and/or diagram to solve the problem.

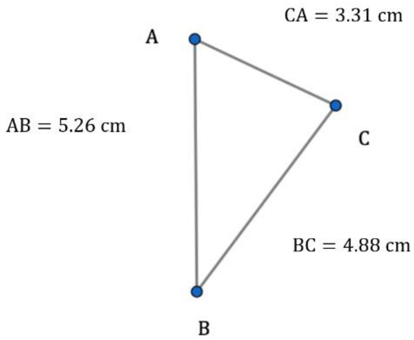
1. Draw an isosceles triangle and an equilateral triangle. Mark the equal sides with tick marks. Measure the sides in centimeters.

2. Draw a scalene triangle. Label it with tick marks to show its sides are not equal.

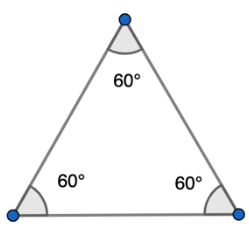
3. Identify the triangle as scalene, isosceles, or equilateral. Justify your answer.



4. Identify the triangle as scalene, isosceles, or equilateral. Justify your answer.

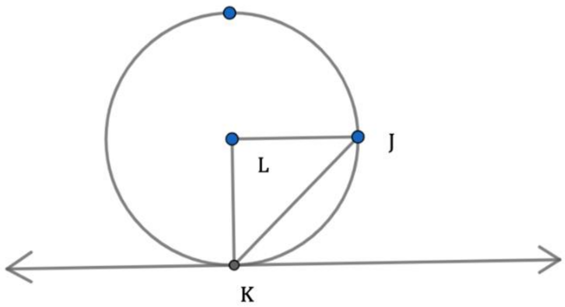


5. Identify the triangle as scalene, isosceles, or equilateral. Justify your answer.



6. How did you justify what kind of triangle was in Problem 5? What do you notice about the angles in an equilateral triangle?

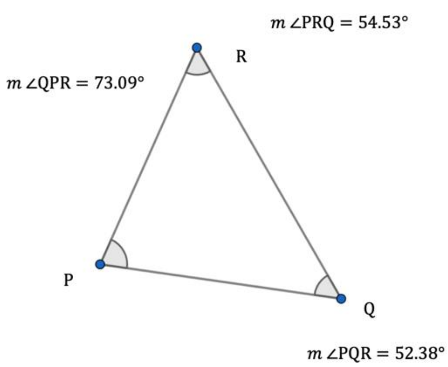
7. In the diagram below, name the type of triangle by its sides (which are located inside the circle). Justify your answer without measuring. Radius LJ is parallel to the tangent line to the circle at point K.



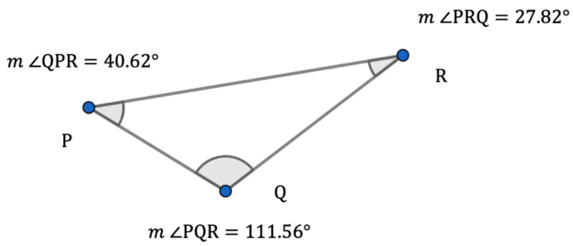
8. How do you think you could use a compass to draw an equilateral triangle in the circle from Problem 7 so that side KJ is equal to KL and LJ?

For Problem 9-12, identify the angles as acute, obtuse, or right. Justify your answer.

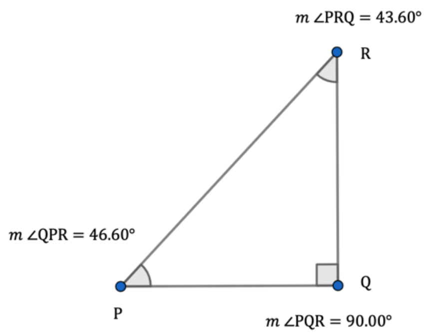
9.



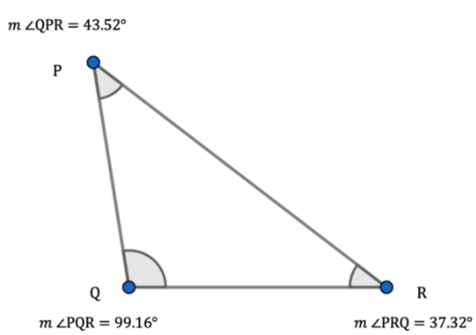
10.


m ∠PRQ = 27.82°
m ∠QPR = 40.62°
R
P
Q
m ∠PQR = 111.56°

11.

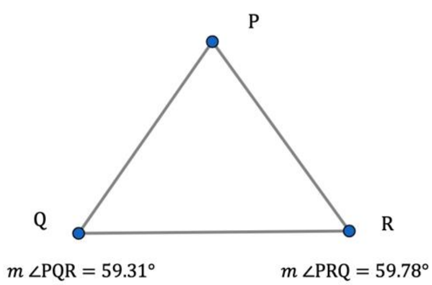

m ∠PRQ = 43.60°
R
m ∠QPR = 46.60°
Q
P
m ∠PQR = 90.00°

12.

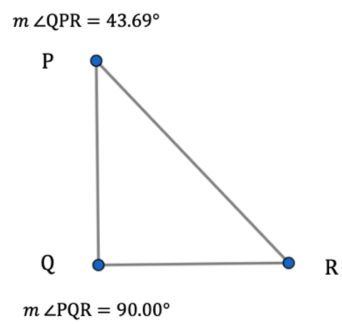

m ∠QPR = 43.52°
P
Q
R
m ∠PQR = 99.16°
m ∠PRQ = 37.32°

For Problem 13 and 14, name the triangles by their angles.

13.

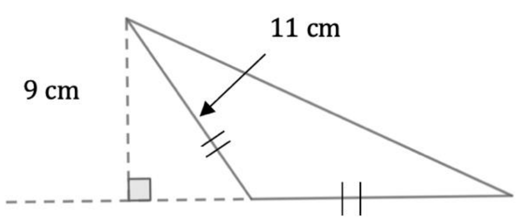


14.



For Problem 15 and 16, find the approximate area of the triangle (the height of the obtuse triangle is outside of the triangle and the height of the acute triangle is inside the triangle). Round to the hundredths place.

15.



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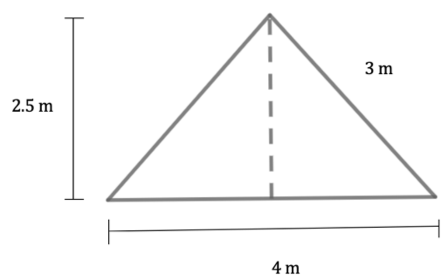
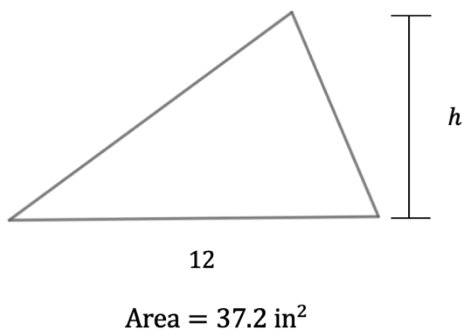
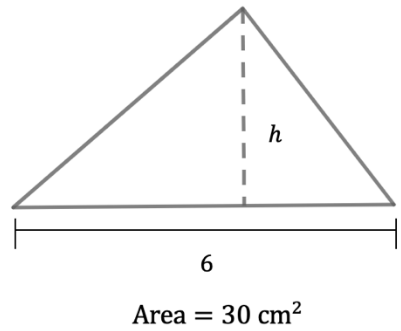


For Problem 17 and 18, find the height of the triangle given.

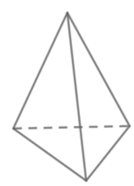
17.



18.



For Problem 19 and 20, use the figure below and information given to solve the problem.



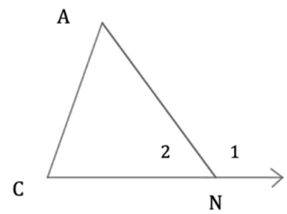
19. Find the volume of the triangular pyramid given the information about its base: b $= 6$ cm; $h = 4$ cm. The height of the pyramid is H $= 18$ cm. Use V = $\frac{1}{3}$BH.

20. The triangular pyramid from Problem 19 becomes the cupholder for an ice cream soda. The cream soda is filled to a height of 13 cm. and when the ice cream is added it rises to 17 cm. What is the volume of the scoop of ice cream added?

Section 6.2 Congruence Postulates/Theorems

Practice Problems 6.2

For Problem 1-5, use the diagram below to solve the problem.



1. Complete the reasons to prove that $m\angle 1 = m\angle C + m\angle A$.

Statement	Reason
1. $m\angle 1 + m\angle 2 = 180°$	1.
2. $m\angle 2 + m\angle A + m\angle C = 180°$	2.
3. $m\angle 2 = 180° - m\angle 1$	3.
4. $180° - m\angle 1 + m\angle A + \angle C = 180°$	4.
5. $m\angle A + m\angle C = m\angle 1$	5.
6. $m\angle C + m\angle A = m\angle 1$	6.

2. Angles that form a linear pair with the interior angles of a triangle are called exterior angles. Angle 2 is an interior angle and angle 1 is an exterior angle. If $m\angle 2 = 42°$, what is $m\angle 1$?

3. The proof above is called the Exterior Angle Theorem. Complete the blanks for the theorem. The measure of the _____________________ angle of a triangle is equal to the sum of the measures of the two nonadjacent _____________________ angles.

4. If $m\angle 1 = 127°$ and $m\angle C = 43°$, what is $m\angle A$?

5. If $m\angle 1$ is $10°$ less than twice the measure of $\angle$ A and the measure of $\angle$C is $63°$, find the other two interior angles of the triangle?

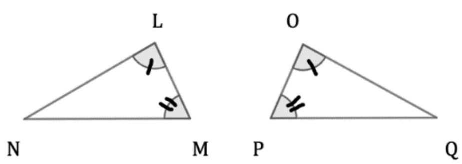
For Problem 6-10, follow the instructions to solve the problem.

6. If ΔABC is a triangle with right angle B and $m\angle \mathrm{A} = 32°$, what is the measure of angle C?

7. Since the acute angles in a right triangle have a sum of $90°$, the acute angles in a right triangle are ____________________.

8. If two right triangles have hypotenuses that are congruent and one leg of each is congruent, then the triangles are congruent by the ____________________-__________ Congruence Theorem.

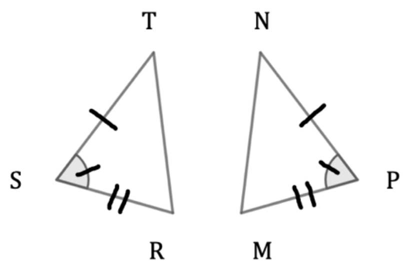
9. Write a proof to show that $\angle N \cong \angle Q$ given that $\angle L \cong \angle O$ and $\angle M \cong \angle P$.



10. Write the theorem from Problem 9, which is sometimes called the Third Angle Theorem.

For Problem 11-14, name the theorem used to prove the pair of angles are congruent.

11.



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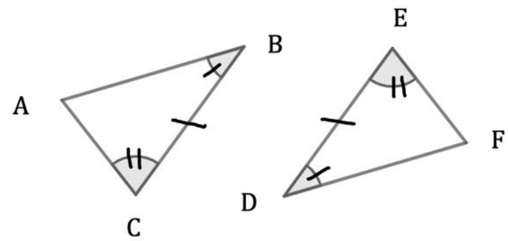

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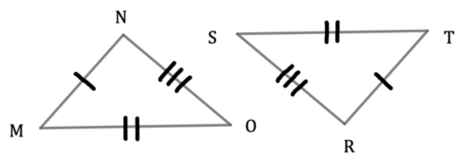

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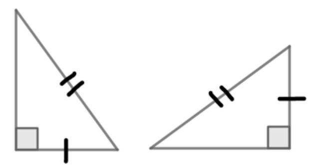
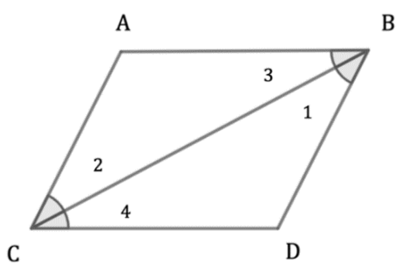


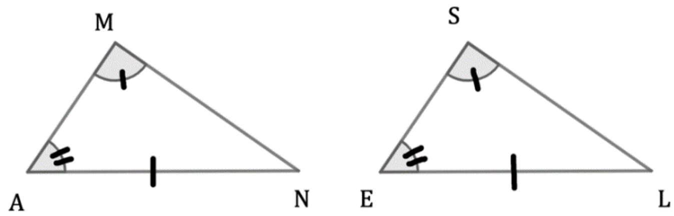
For Problem 15 and 16, follow the instructions to solve the problem.

15. Find the error in the proof below given ABCD is a parallelogram.



Statements	Reasons
1. ABCD is a parallelogram	1. Given
2. $\overline{AB} \parallel \overline{CD}$	2. Definition of a Parallelogram
3. $\overline{AB} \cong \overline{CD}$	3. Definition of a Parallelogram
4. $\angle 1 \cong \angle 4$	4. Alternate Interior Angles Congruence Theorem
5. $\angle 2 \cong \angle 3$	5. Alternate Interior Angles Congruence Theorem
6. $\Delta ABC \cong \Delta DCB$	6. AAS Postulate or SAA Theorem

16. Fill in the blanks for ΔMAN and ΔSEL.



$\angle M \cong$ ________ $180° - ($________ $+$ ________$) = m\angle N$

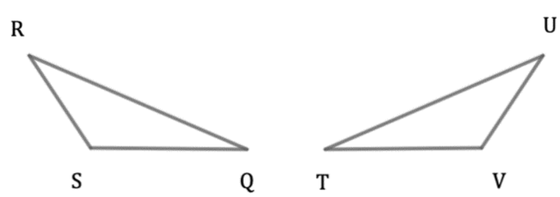
$\angle A \cong$ ________ $180° - ($________ $+$ ________$) = m\angle L$

$\overline{AN} \cong$ ________ $\angle N \cong$ ________

$\Delta MAN \cong$ ________

by __________ or __________

For Problem 17 and 18, given $\overline{RQ} \cong \overline{UT}$ and $\angle Q \cong \angle T$, what other given is needed to prove that $\Delta RQS \cong \Delta UTV$ using the stated theorem.

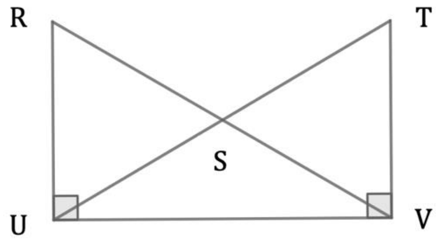


17. SAS Congruence Theorem

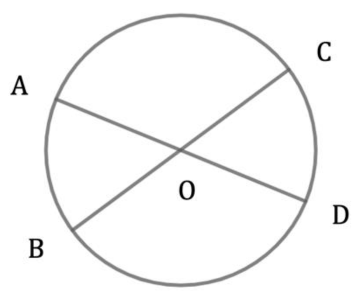
18. ASA Congruence Theorem

For Problem 19 and 20, follow the given instructions to solve the problem.

19. If $\overline{RU} \cong \overline{TV}$, what else is needed to use the HL Congruence Theorem to prove that $\Delta RVU \cong \Delta TUV$?



20. Prove that $\Delta AOB \cong \Delta COD$ given circle O.

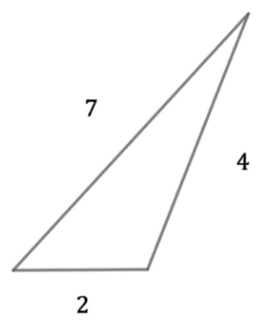


Section 6.3 Corresponding Parts of Congruent Triangles

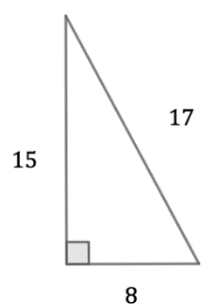
Practice Problems 6.3

For Problem 1-3, tell if the triangle is possible and explain why or why not.

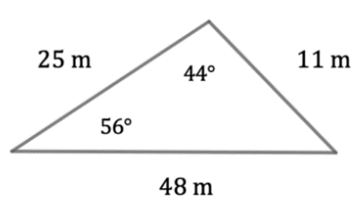
1.



2.

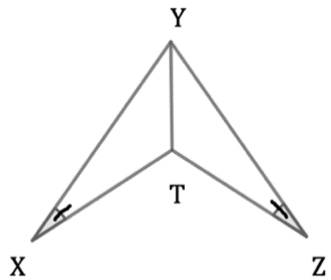


3.

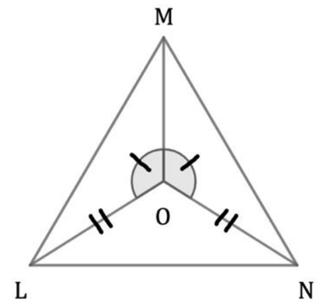


For Problem 4-7, name the congruent angles and sides in the congruent triangles.

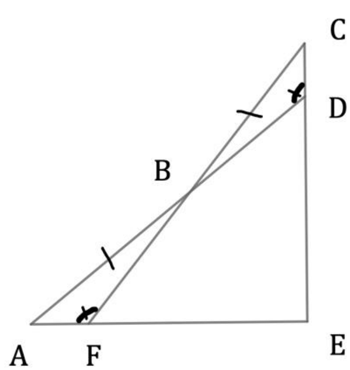
4. $\Delta XYT \cong \Delta YZT$



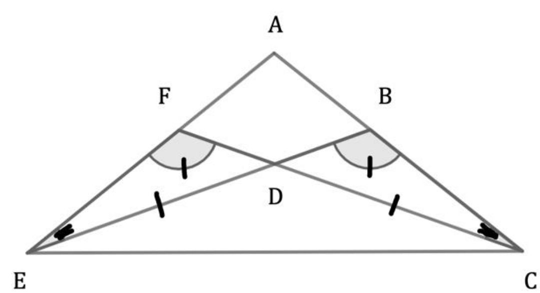
5. $\Delta MOL \cong \Delta MON$



6. $\Delta AFB \cong \Delta CDB$

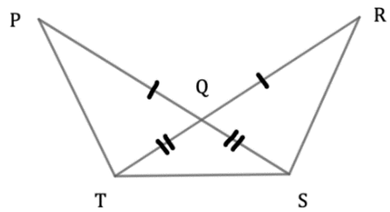


7. $\Delta FDE \cong \Delta BDC$

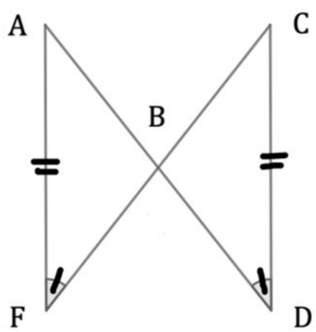


For Problem 8-9, using the given segments and angles marked congruent, tell what congruence theorem can be used along with one more piece of information to prove that the triangles are congruent.

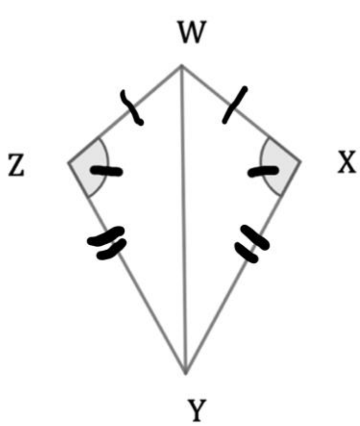
8. $\Delta PQT \cong \Delta RQS$



9. $\Delta FBA \cong \Delta DBC$



10. What two congruence theorems prove that $\Delta ZWY \cong \Delta XWY$?



For Problem 11-20, follow the instructions to solve the problem.

11. What is a Congruence Theorem that would demonstrate that triangles are rigid polygons?

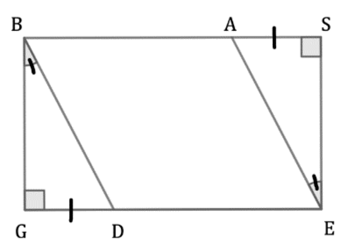
12. A parallelogram is not a rigid structure. What can be done to make it a rigid structure?

13. Complete the table for the number of diagonals needed to make the polygons rigid structures.

Number of Sides	3	4	5	6	7	8
Number of Diagonals						

14. If n is the number of sides for the polygon, what is the number of diagonals needed to make the polygon rigid in terms of n.

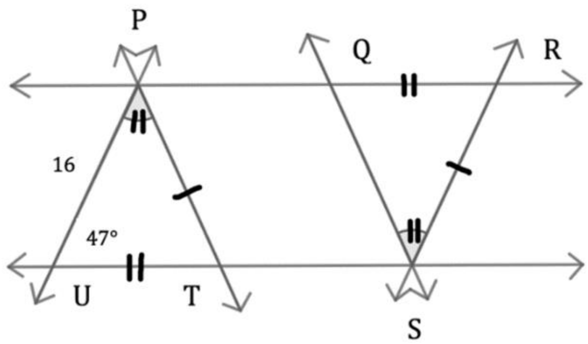
15. Fill in the blank: Given rectangle BSEG, $m\angle \text{GBD} = 43°$, and $m\angle \text{EAS} =$ ________.



16. What other angle in Problem 15 is congruent to ∠EAS?

17. In a formal proof, what Congruence Theorem(s) demonstrate(s) $\Delta \text{ESA} \cong \Delta \text{BGD}$?

18. Fill in the blank using the diagram:



$\Delta TPU \cong \Delta RSQ$

$\overline{SR} \cong$ ________

19. In Problem 18, ∠UPT is 66°. What is the measure of ∠QRS?

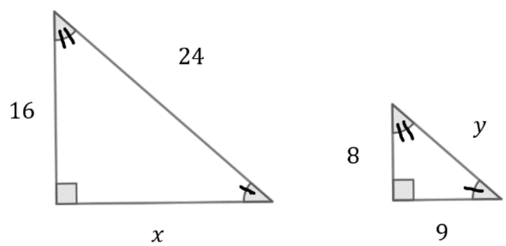
20. What is the measure of QS?

Section 6.4 Similar Triangles

Practice Problems 6.4

For Problem 1-3, find the scale factor and missing side lengths of the similar triangles.

1.



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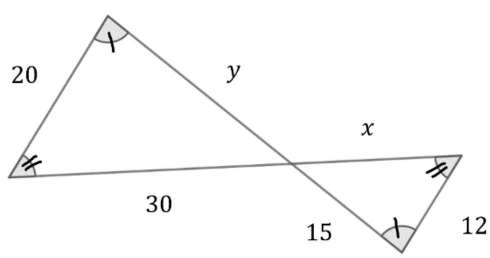
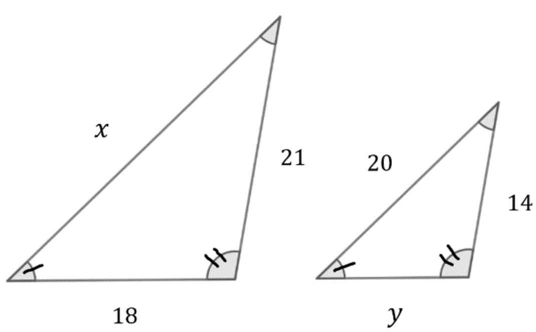
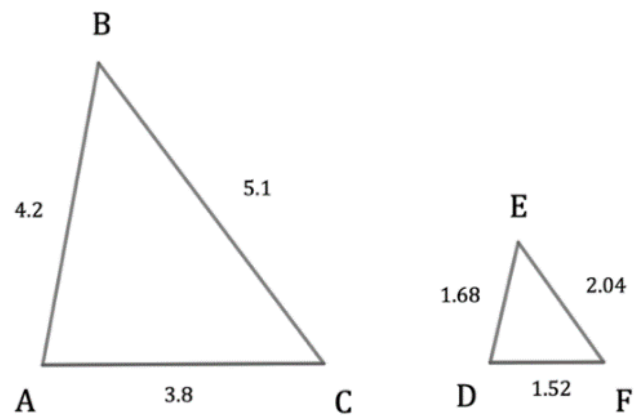


3.

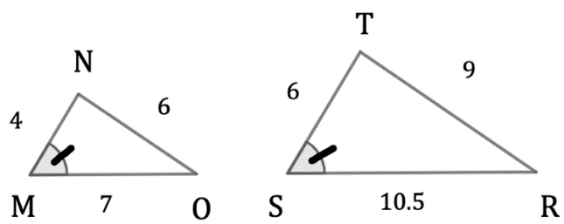


For Problem 4-6, use the triangle(s) given to solve the problem.

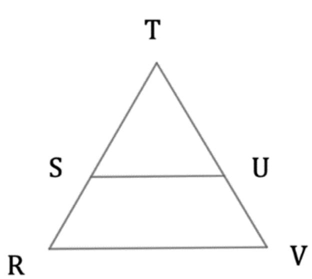
4. Is ΔABC~ΔDEF? Explain your reasoning.



5. The triangles GHI and JKL are similar. Name all pairs of congruent angles.



6. Given $\overline{SU} \parallel \overline{RV}$, show that ΔSTU~ΔRTV.



For Problem 7-15, follow the instructions to solve the problem.

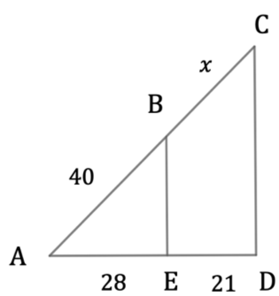
7. Fill in the Blank: Since the triangles in Problem 6 are similar, $\frac{\text{TS}}{\text{SR}} =$ ________.

8. Fill in the Blanks: The Triangle Proportionality Theorem states that if a line is parallel to one side of a triangle, then it ____________________ the other two sides, and it divides the two sides ____________________.

9. Write the Converse of the Triangle Proportionality Theorem.

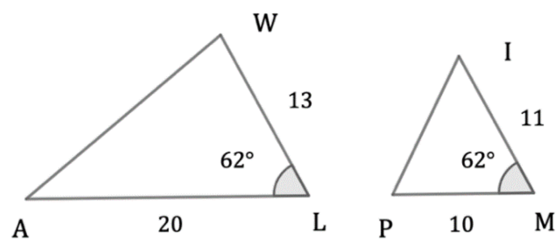
10. Fill in the Blank: In Problem 6, since $\frac{\text{TS}}{\text{SR}} = \frac{\text{TU}}{\text{UV}}$, then $\overline{\text{SU}}$ is ____________________ to $\overline{\text{RV}}$.

11. Given $\overline{\text{BE}} \parallel \overline{\text{CD}}$, find the length of $\overline{\text{BC}}$.

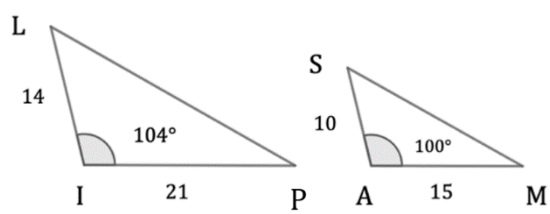


12. Write the contrapositive of the Triangle Proportionality Theorem using the side lengths in Problem 11.

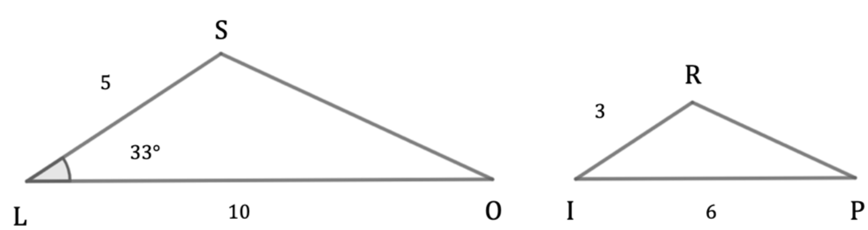
13. Is ΔAWL~ΔPIM?



14. Is ΔLIP~ΔSAM?

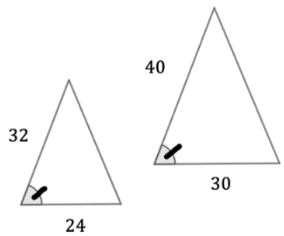


15. If ΔSLO~ΔRIP, what is the measure of ∠I?



For Problem 16-18, name the Similarity Theorem that proves the triangles are similar.

16.



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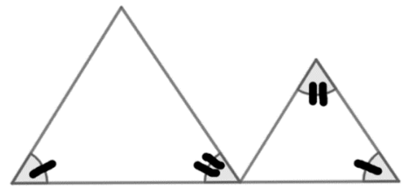

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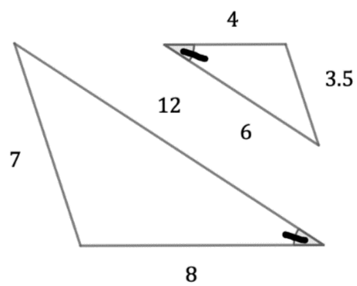
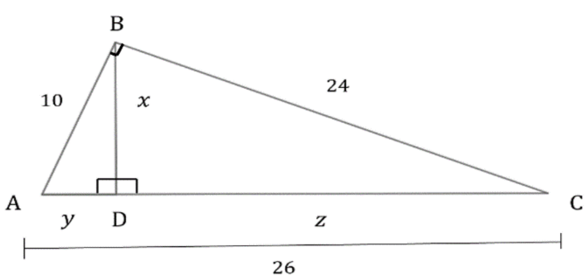
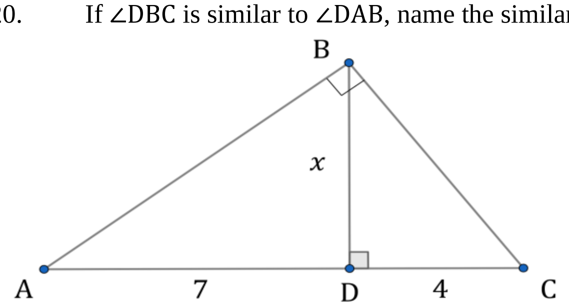


For Problem 19 and 20, use the triangle given to solve the problem.

19. If ΔABC~ΔADB~ΔBDC find the length of AB, BD and DC. Round to the nearest tenth.



20. If ∠DBC is similar to ∠DAB, name the similar triangles and find the value of x.

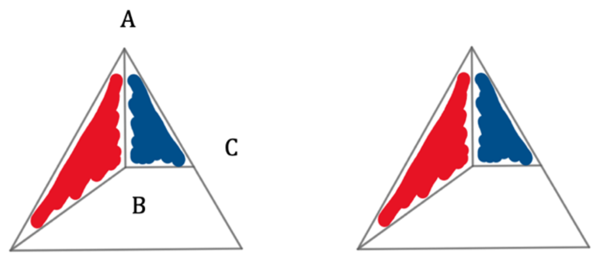


Section 6.5 Transformations of Triangles

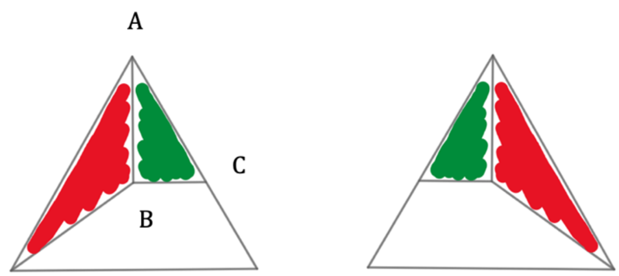
Practice Problems 6.5

For Problem 1-3, use the triangles given to solve the problem.

1. Find the points A′, B′, and C′ that are located on the image and are horizontal translations of points A, B, and C.



2. Find the points A′, B′, and C′ that are reflections in the y-axis from the pre-image points A, B, and C.



3. Given points $\mathrm{A}(0, 4)$, $\mathrm{B}(0, 0)$, and $\mathrm{C}(3, 0)$ for the triangles from Problem 2, name the coordinates for A′, B′, and C′ for a reflection in the y-axis.

For Problem 4-7, use the information given to solve the problem.

4. Which coordinate transformation represents a reflection in the y-axis?

a) $(x, y) \rightarrow (x, -y)$ b) $(x, y) \rightarrow (-x, y)$

c) $(x, y) \rightarrow (y, x)$ d) $(x, y) \rightarrow (-x, -y)$

5. Using the answer from Problem 4, what coordinate transformation represents a reflection in the x-axis?

6. Given the coordinates from Problem 3, if ΔABC were reflected in the x-axis, what would be the image coordinates for A′, B′, and C′?

7. Given the transformation $(x, y) \rightarrow (y, x)$ and ΔABC with coordinates A$(-3, 1)$, B$(-5, 4)$, and C$(-3, 7)$, write the coordinates for A′, B′, and C' for the image of the triangle.

8. What is the line of the reflection of symmetry for Problem 7?

For Problem 9-13, tell whether the statement is true or false.

9. The perpendicular bisector of every segment connecting a point in the pre-image to its corresponding point in the image is the line of reflection.

10. If a figure coincides with every point under an isometry, then the figure has symmetry.

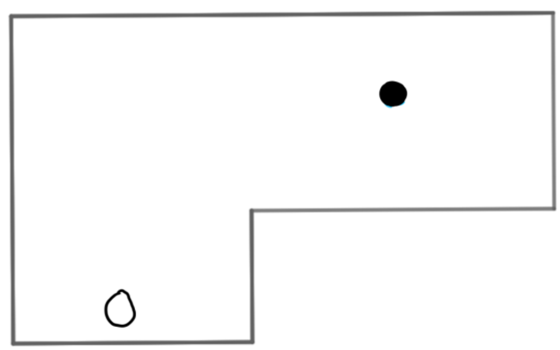
11. Two reflections across a pair of intersecting lines is the same as a single rotation where the angle of rotation is twice the acute angles formed by the intersecting line.

12. The transformation $(x, y) \rightarrow (y, x)$ is the same as the two transformations $(x, y) \rightarrow (x, -y)$ and $(x, y) \rightarrow (-x, y)$ performed consecutively.

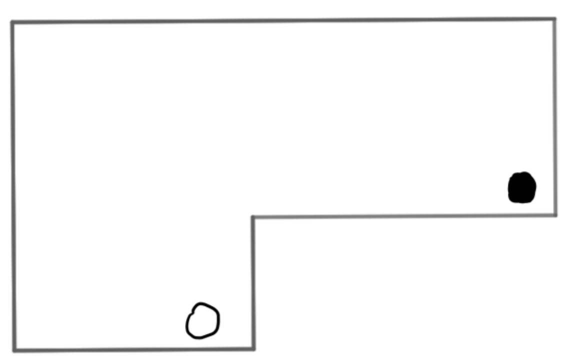
13. The simplest type of isometry is a rotation.

For Problem 14 and 15, follow the instructions to solve the problem.

14. Place an “x” where you will hit the putt-putt ball to make a hole in one. The open circle is the tee, and the closed circle is the hole.

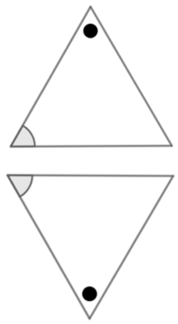


15. If you reflect the hole over the line above it, the ball will hit the corner. Reflect the hole over the line below it, then reflect that over the top line and mark an “x” on the top line where you will hit the golf ball to get a hole in one on the putt-putt course.

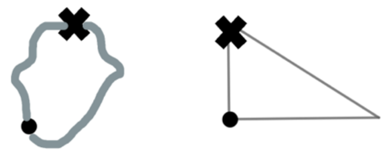


For Problem 16-19, tell whether the transformations are translations, reflections, rotations, or non-rigid.

16.



17.



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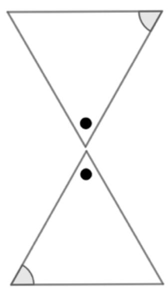
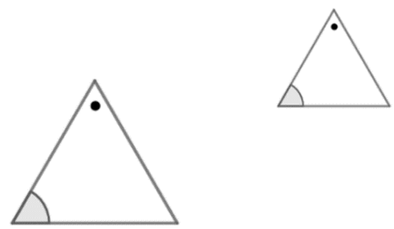
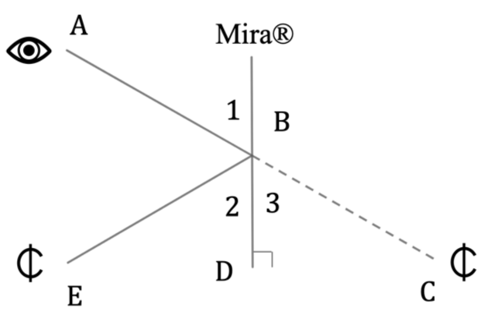


19.



For Problem 20, prove that the distance from the penny to the Mira® is equal to the distance from the penny's reflection to the Mira®, $EB = CB$ using the information and diagram given.

20. When you reflected a penny using a Mira®, you saw the penny bouncing off the light. The incoming angle from your eyes to the Mira® is congruent to the outgoing angle from the Mira® to the penny; $\angle 1 \cong \angle 2$ because the incoming angle is equal to the outgoing angle.

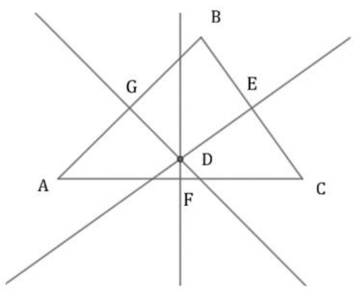


Section 6.6 Circumcenter and Incenter

Practice Problems 6.6

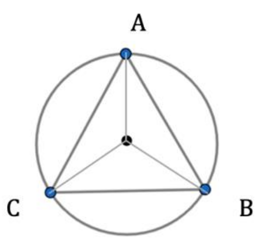
For Problem 1-20, follow the instructions given to solve the problem.

1. Write a formal proof to prove that $\overline{DA} \cong \overline{DC}$ given that D is the circumcenter of triangle ABC.



Statement	Reason
1. D is the circumcenter of ΔABC	
2. $\overline{DF} \perp \overline{AC}$ and $\overline{DF}$ bisects $\overline{AC}$	
3. $\angle DFA$ and $\angle DFC$ are right angles	
4. $\angle DFA \cong \angle DFC$	
5. $\overline{DF} \cong \overline{DF}$	
6. $\overline{AF} \cong \overline{FC}$	
7. $\Delta AFD \cong \Delta CFD$	
8. $\overline{DA} \cong \overline{DC}$	

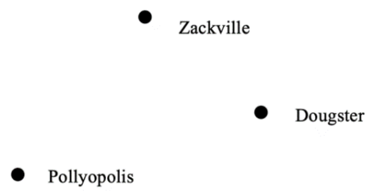
2. Use a compass and open it the distance from the circumcenter of the triangle ABC to one of the vertices. What do you notice at each vertex?



3. Is the circle in Problem 2 circumscribed about the triangle or inscribed about the triangle?

4. Which parts of the circle are the segments from the circumcenter to each vertex?

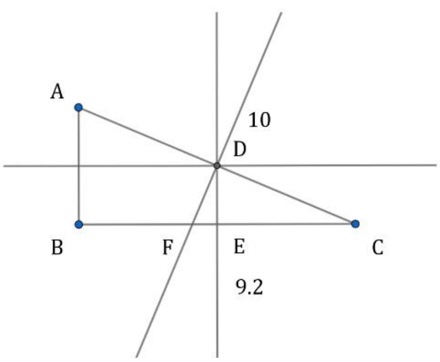
5. An organic farm wants to locate an equal distance to all three farmers' markets in the three suburbs where it sells produce (Zacksville, Dougster, and Pollyopolis). Where should the organic farm purchase property to grow its crop?



6. Draw an obtuse triangle and find the circumcenter. Use the compass and protractor of a geometer's utility tool. Where is the circumcenter located?

7. Construct a right triangle or draw it with a protractor. Use a geometer's utility tool. Find the circumcenter; where is it located?

8. In triangle ABC, $\mathrm{AC} = 10$ and $\mathrm{BC} = 9.2$. Given the shortest distance between a point and a line is the perpendicular bisector, can $\mathrm{FD} = 1.6$? Let point D be the circumcenter of right triangle ABC.



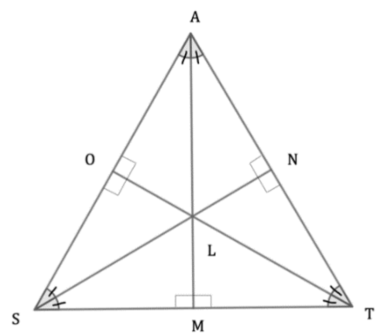
9. Find the circumcenter of a triangle with coordinates $\mathrm{A}(-4, 2)$, $\mathrm{B}(0, -4)$, and $\mathrm{C}(3, 2)$.

10. Find the circumcenter of a right triangle with coordinates $A(0,2)$, $B(0,-4)$, and $C(2,2)$.

11. What did you notice about the legs of the right triangle from Problem 10 that made finding the circumcenter easier?

12. Write a proof for ΔSAT.

Given ΔSAT, $\overline{SL}$ bisects ∠AST, $\overline{LO} \perp \overline{SA}$, and $\overline{LM} \perp \overline{ST}$, prove $\overline{LO} \cong \overline{LM}$.



Statement	Reason

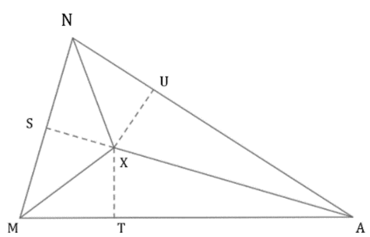
13. Find the incenter of a right triangle. Where is it located?

14. Find the incenter of an obtuse triangle. Where is it located?

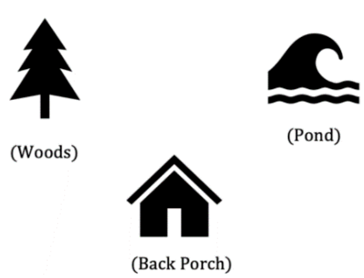
15. What can you say about the location of all incenters of triangles?

16. Use a compass or geometer's utility to draw a circle from the incenter of the triangle so that the radius is the length of the perpendicular line to the side. Is the circle inside or outside the triangle? Is the circle inscribed in the triangle or circumscribed about the circle? Is the triangle inscribed in the circle or circumscribed about the circle?

17. In the triangle below, $\overline{MX}$, $\overline{NX}$, and $\overline{AX}$ are angle bisectors and $\overline{XS}$, $\overline{XT}$, and $\overline{XU}$ are the line segments from incenter X perpendicular to each side. If $XT = 9$ and $SM = 12$, find the length of XS and MX.



18. A family wants to build a fire pit that is the center piece of their backyard, which will be between their back porch, pond, and the woods. How can the center be located?



Section 6.7 Orthocenter and Centroid

Practice Problems 6.7

For Problem 1-9, follow the steps to find the orthocenter of a triangle GOD with coordinates $G(6,0)$, $O(8,5)$, and $D(6,7)$.

1. Use graph paper to graph points G, O, and D. Connect the points in order to make triangle GOD.

2. What is the only altitude inside the triangle? Draw the altitude on the graph paper. What is the slope? What is the equation of the altitude?

3. Which side is perpendicular to the altitude inside the triangle. What is the slope? What is the equation for the perpendicular side?

4. Draw the altitude from point D. Where is the altitude located?

5. Find the slope for line segment GO?

6. What is the slope for a line perpendicular to GO?

7. Use point D in the point-slope formula and the slope from Problem 6 to find the equation of the line perpendicular to GO through point D. Solve for x in terms of y.

8. Find the intersection of the altitudes from point O and point D to their opposite sides by solving the system of equations from Problem 2 and Problem 7.

9. Use a graphing utility to find the orthocenter of triangle GOD. Is it the same point you found in Problem 9?

For Problem 10-13, fill in the blank.

10. In a right triangle, the altitudes from each vertex at the ends of the hypotenuse are the _____________________ of the triangle.

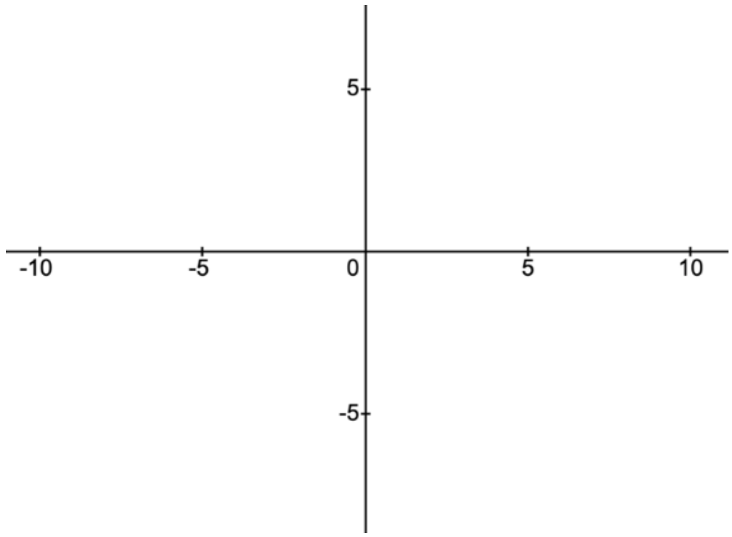
11. The medians of a right, acute, and obtuse triangle all lie _____________________ of the triangle.

12. The third side of a triangle that is parallel to the midsegment is _____________________ the length of the midsegment.

13. The midsegments of a triangle form _____________________ congruent triangles.

For Problem 14-20, use the triangle with coordinates $A(0,4)$, $B(0,-4)$, and $C(8,0)$ to solve the problem.

14. Sketch a graph of the triangle.



15. a) What is the equation of the altitude from vertex C?

b) What is the equation of the median from vertex C?

16. a) What can be said about the median and altitude of an isosceles triangle?

b) How do you know ΔABC is isosceles?

17. a) What is the midpoint of $\overline{AC}$?

b) What is the midpoint of $\overline{BC}$?

18. How long is the midsegment that is parallel to $\overline{AB}$?

19. What is the equation of the median from vertex A to $\overline{BC}$?

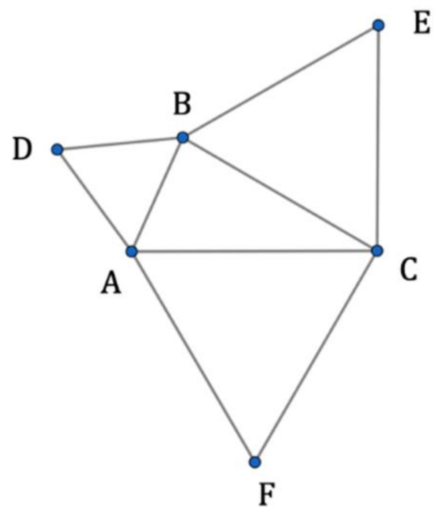
20. Find the centroid of ΔABC.

Section 6.8 Napoleon's Theorem

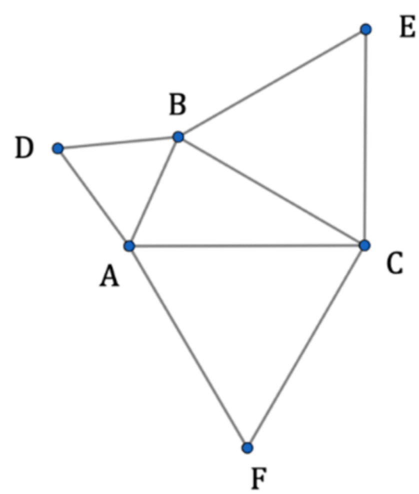
Practice Problems 6.8

For Problem 1-6, follow the instructions to solve the problem.

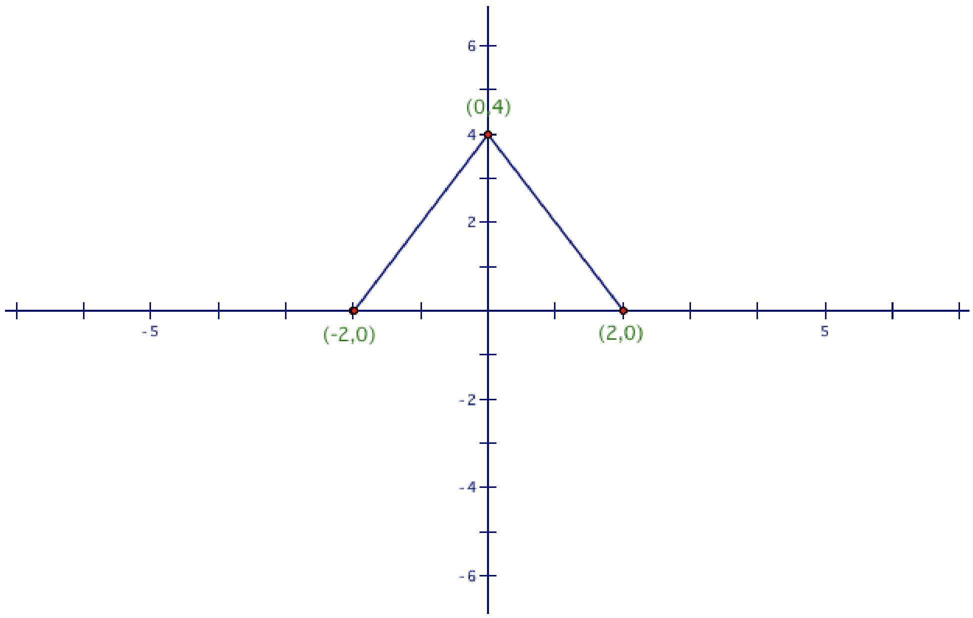
1. Construct an inner Napoleon triangle. In order to do this, use a Mira® along each side of the original triangle and reflect the centroids of the equilateral triangles over the corresponding side in the original triangle, then connect the reflections of the centroids. What do you notice about the inner Napoleon triangle?



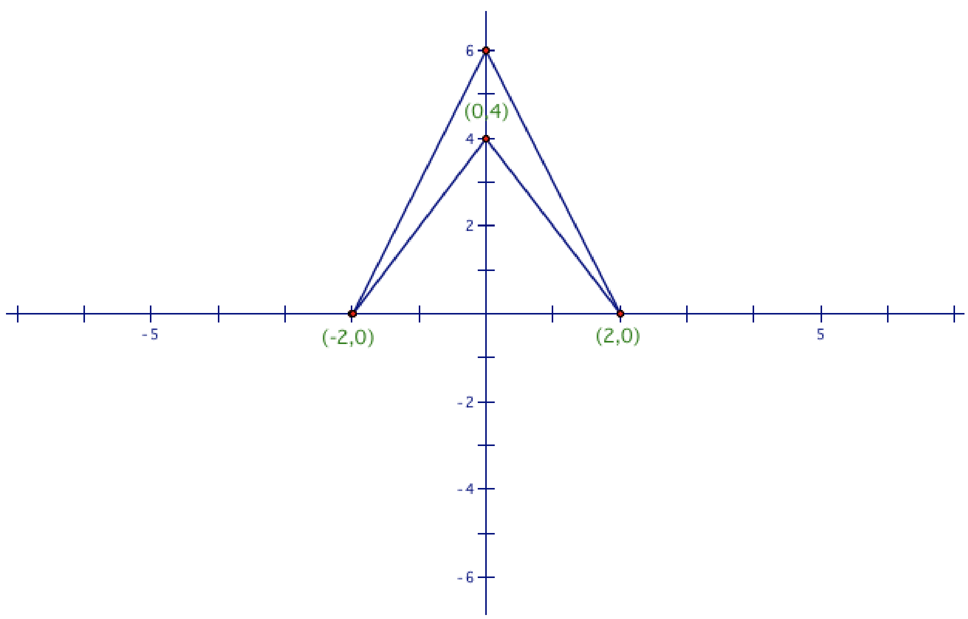
2. Construct segments by connecting each vertex A, B, and C of the original triangle in Problem 1 with the vertex of the equilateral triangle created on the opposite side of A, B, and C. What do you notice about these segments?



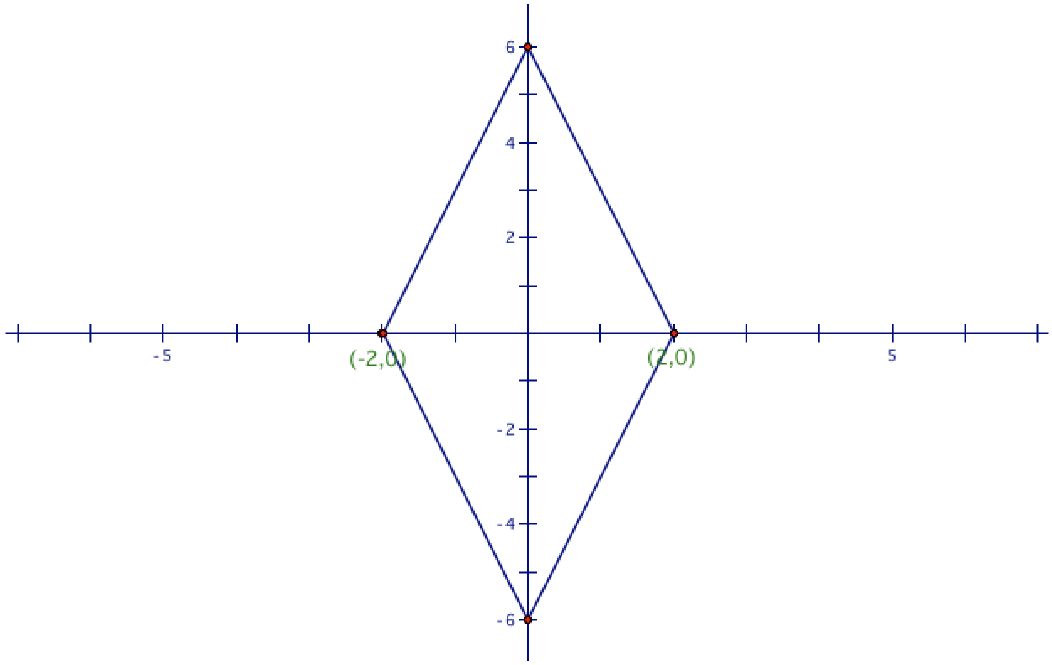
3. Using graph paper, draw the lines $y = 2x + 4$ and $y = -2x + 4$. Restrict the domain for the first equation to $-2 \leq x \leq 0$ and for the second equation to $0 \leq x \leq 2$. Color in a triangle bounded by the line segments of the restricted domain and the x-axis. What type of triangle is this? Name the coordinates of all three vertices.



4. Write the two equations for the lines that would create a triangle that has the top vertex at point $(0, 6)$ but has the same x-intercepts and domain restrictions as the previous triangle in Problem 3. What are the equations for the two sides of the isosceles triangle?



5. If you reflected the triangle in Problem 4 over the x-axis, what would the equations of the lines be that are below the x-axis? What are these x-intercepts? What are the domain restrictions that create the line segments of the sides of the triangle?



6. What is the centroid of the smaller triangle in Problem 3?

7. What is the centroid of the larger triangles in Problem 4 and Problem 5?

8. Besides the triangles in Problem 4 and Problem 5, write equations for other lines above and below the x-axis that would create isosceles triangles with an altitude that is a median on the y-axis. What rules could you describe?

Section 6.9 Fractals and the Sierpinski Triangle

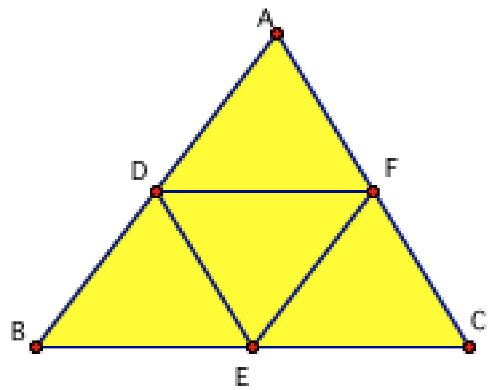
Practice Problems 6.9

1. Follow the steps below to draw a Sierpinski triangle, which is a fractal.

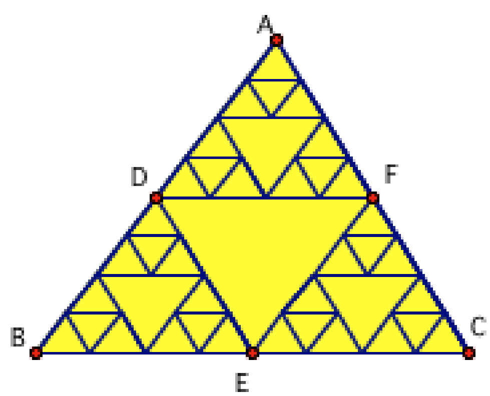
Step 1: Draw triangle ABC and find the midpoints of each of the three sides.

Step 2: Label the midpoints D, E, and F on sides AB, BC, and CA.

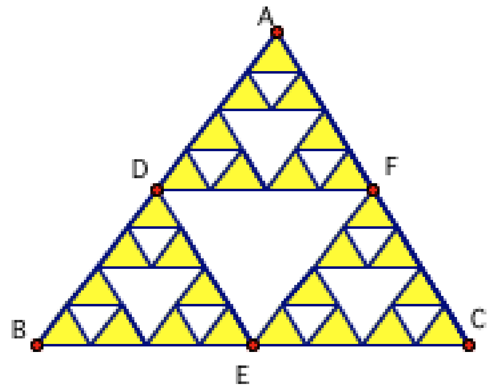
Step 3: Construct four smaller equilateral triangles in the larger triangle by connecting vertices.



Step 4: Find the midpoints of each of the sides of the new triangles and make those points the vertices of a new and smaller equilateral triangle.

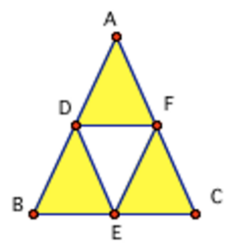


Step 5: Keep iterating the process to form triangles within triangles. Use color on the interior of the triangles to highlight patterns. Do not color the triangle formed on the interior of the medians.

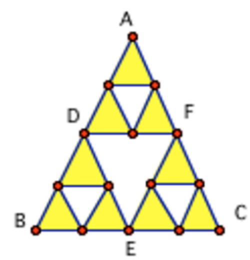


For Problem 2, use the information and diagram below to complete the table.

2. Let the initial triangle be Stage 0 when the whole triangle is colored yellow and has an area of 1 unit as seen in the table. At each stage, color the newly created triangles yellow and find their number and area to complete the table. Stage 1 and Stage 2 are shown below. Stage 3 is shown in Step above in Problem 1.



Stage 1

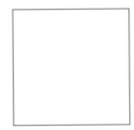


Stage 2

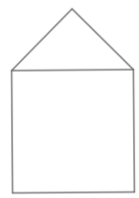
Stage Number	0	1	2	3	4	5	6	*n*
Triangles Colored Yellow	1	3		27				
Area of Colored Triangles	1	$\frac{3}{4}$		$\frac{27}{64}$				

3. Follow the steps below to draw a Pythagorean Tree which is a fractal formed by squares and triangles.

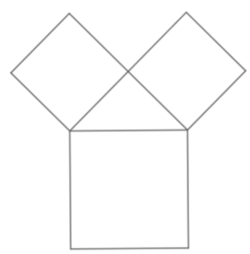
Step 1: Draw a 1″ by 1″ square at the bottom center of your page.



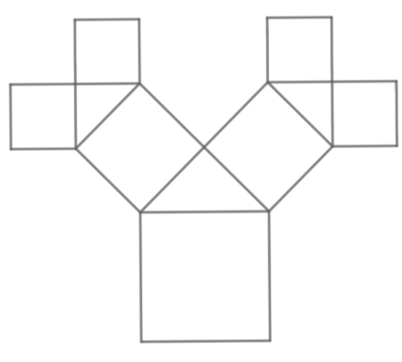
Step 2: Draw a right isosceles triangle on the top of the square so that the top of the square is the hypotenuse of the isosceles right triangle. This will look like a roof on a house.



Step 3: Draw squares on each leg of the isosceles triangle that is the roof of the house. These will look like the branches of a tree.



Step 4: Repeat the pattern on each branch of the tree. Each iteration will get continually smaller until the page is full. Look up a Pythagorean tree to compare it to your tree when you are done. Why do you think this is called a Pythagorean tree?



Section 6.10 Right Triangles and the Pythagorean Theorem

Practice Problems 6.10

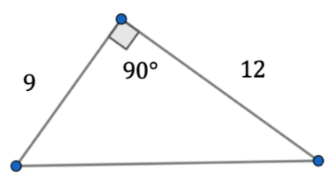
For Problem 1 and 2, use the Pythagorean Theorem to solve the problem.

1. Cintia is washing windows in her 2-story house. If she puts the 18-foot ladder 4 feet from the house to get the ladder right underneath the window, how high up from the ground is the window?

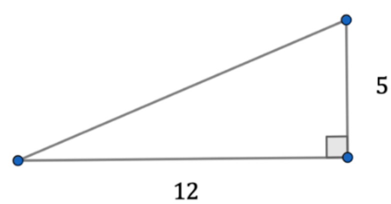
2. A rectangular fence is to be built in a space 5 meters wide. A dog is tied in one corner of the space and his rope is 12 meters long so he can run to the other corner along the diagonal. What amount of fence will need to be purchased for the entire perimeter?

For Problem 3 and 4, use the Pythagorean Theorem to find the missing side of the triangle given.

3.

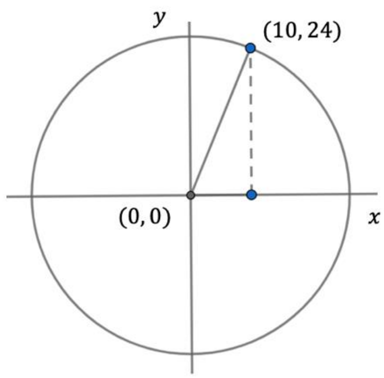


4.

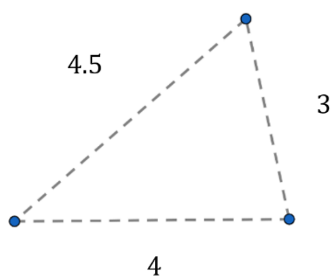


For Problem 5 and 6, use the given diagram to solve the problem.

5. What is the length of the radius of the circle?



6. Is the triangle below a right triangle?



For Problem 7-10, use the information below to tell whether the set of three numbers is a Pythagorean Triple. If they are- write true, if not- write false.

The positive integers that work in the equation of the Pythagorean Theorem are called Pythagorean Triples.

7. $16-30-34$

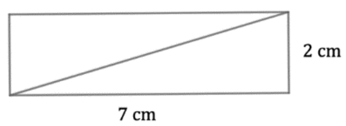
8. $7-24-25$

9. $10-24-26$

10. $12-14-16$

For Problem 11-14, find the diagonal of each rectangle given the lengths of the sides in the given shape.

11.



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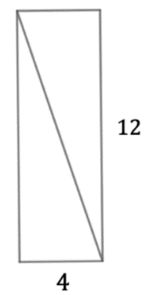

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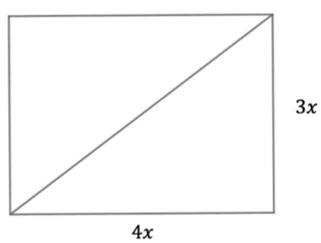

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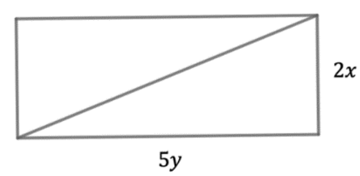
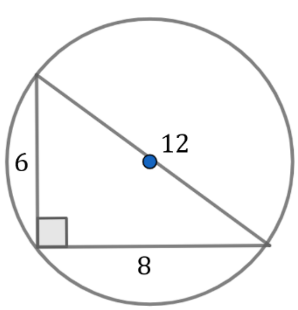


For Problem 15-20, given the information/diagram solve the problem.

15. What is incorrect in the diagram below?

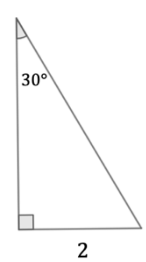


16. The scarecrow in the movie “The Wizard of Oz” says: “In an isosceles triangle, the sum of the square roots of the two equal sides is equal to the third side.” Is this true?

17. What are each of the legs in an isosceles right triangle multiplied by to get the hypotenuse?

18. What are the lengths of the three sides of a right triangle if they are three consecutive even integers?

19. Find the length of the long leg of the triangle below if the hypotenuse is 4 units.



20. In Problem 19, what do you multiply the short leg by to find the hypotenuse? What do you multiply the short leg by to find the long leg?

Section 6.11 The Converse of the Pythagorean Theorem

Practice Problems 6.11

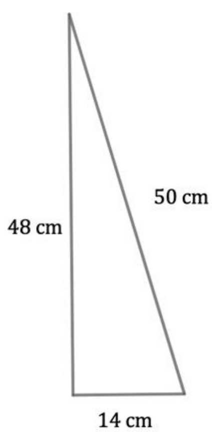
For Problem 1 and 2, use the Pythagorean Theorem to solve the problem.

1. Tamra thinks a window looks like it has square corners. She measures the length, width, and the diagonal from upper right corner to lower left corner; the measurements for the window are 7 decimeters and 24 decimeters, and the diagonal measures 25 decimeters. Does the window have square corners?

2. A right triangle has one leg that is 8 centimeters in length. The hypotenuse measures 17 centimeters. What is the area of the right triangle?

For Problem 3-6, tell whether the triangle is a right triangle or not.

3.



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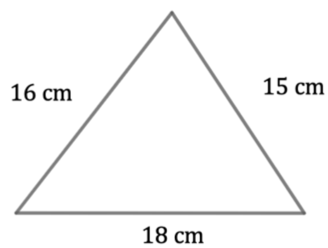

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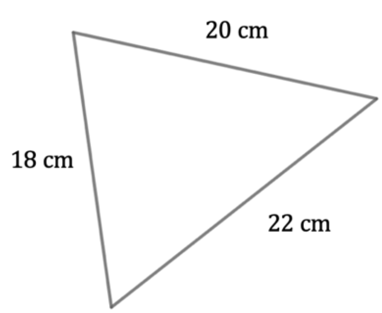

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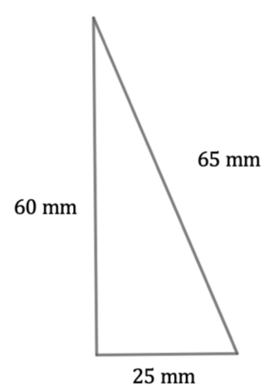
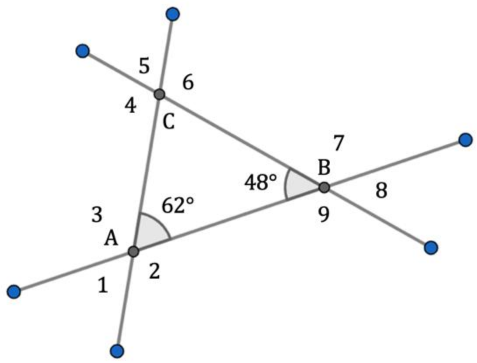


For Problem 7, use the diagram below to solve the problem.



7. a) What is the measure of angle C? How do you know?

b) What is the measure of angle 5? How is it related angle C?

c) What is the measure of angle 6? Is it a complement or supplement to angle 5?

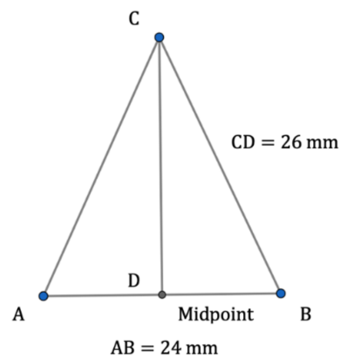
d) Find the measure of angle 4 without measuring or using any calculations.

e) Find the measures of angles 1, 2, and 3.

f) Find the measures of angles 7, 8, and 9.

g) Looking at exterior angle 3, what are the two remote interior angles? What should their sum be? Is this true?

For Problem 8-11, use the isosceles triangle ABC to solve the problem.



8. What is the length of AD? How do you know?

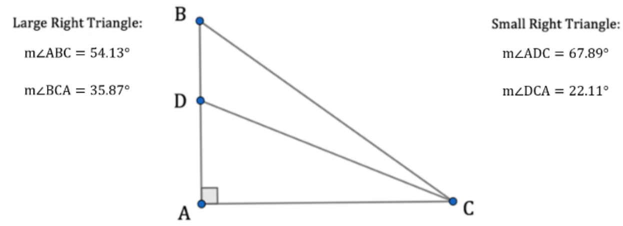
9. Angle CDA is a right angle. What is the measure of angle CDB?

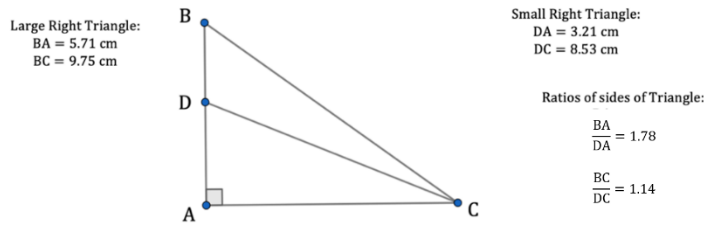
10. If angle ACB is 48°, what is the measure of angle ACD?

11. The perpendicular bisector CD of angle ACB is also an altitude and median of triangle ACB. Explain why.

For Problem 12-14, use the triangles below to solve the problem.

The right triangles share right angle A.



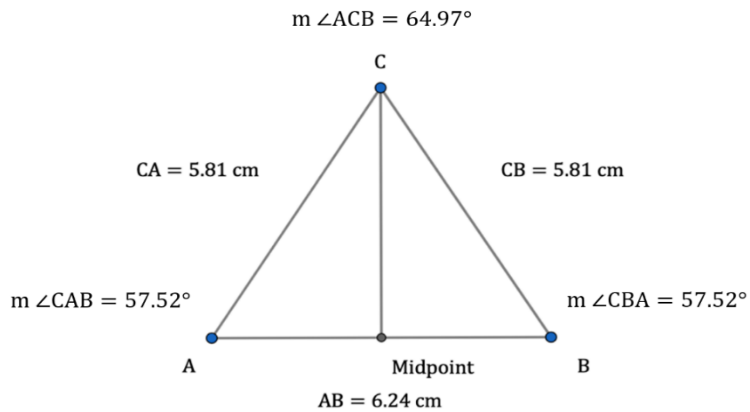


12. Is triangle CAB congruent to triangle CAD?

13. Is triangle CAB similar to triangle CAD?

14. Are all right triangles similar?

For Problem 15-17, use the triangle ABC with extended sides to answer the question.



15. a) What is the name of ΔABC by its sides?

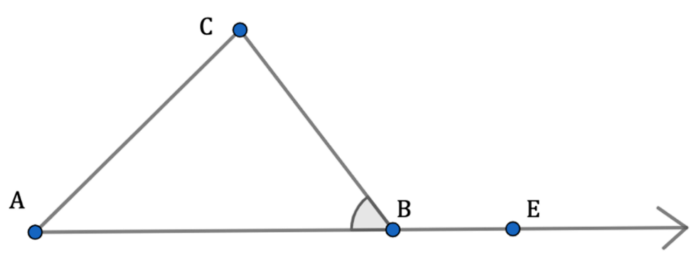
b) What is the name of ΔABC by its angles?

16. What can you say about the base angles of the triangle?

17. Draw a point on the perpendicular bisector from C to the midpoint and connect the point to A and measure the distance. What is the measure of the distance from the same point to B?

For Problem 18-20, use the triangle below to solve the problem.

Angle CBA measures approximately 53°.



18. Name the exterior angle to ∠CBA. What is the measure of this exterior angle?

19. Name the remote interior angles to the exterior angle in Problem 18.

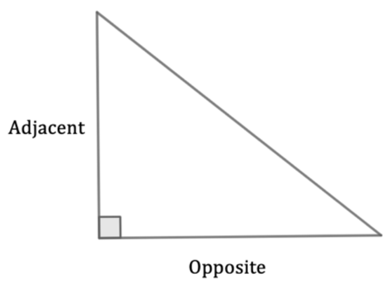
20. What type of angles are ∠CBA and ∠CBE when taken together?

Section 6.12 Special Ratios in Right Triangles

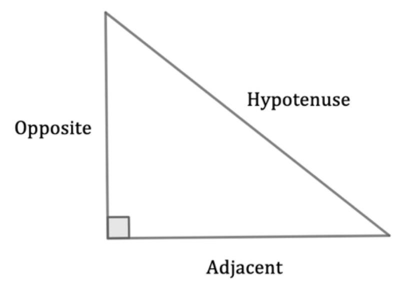
Practice Problems 6.12

For Problem 1 and 2, find θ given the right triangle.

1.



2.



For Problem 3, answer the question.

3. Find the cosine of $60°$. Why is it the same as $\sin 30°$?

For Problem 4-8, fill in the blank(s).

4. The tangent function is a ratio of the ____________________ of a right triangle.

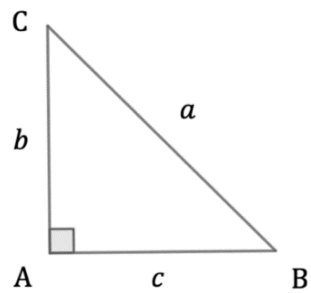
5. The ____________________ and ____________________ functions are ratios of the length of one leg to the hypotenuse.

6. The ____________________ side of the sine function is the same side as the opposite side for the cosine function in a right triangle.

7. The adjacent side of a $60°$ angle is the same as the ____________________ side of a $30°$ angle for the cosine function.

8. If $\sin 45° \approx 0.707$, then $\cos 45° \approx$ __________.

For Problem 9-12, use the variable names of the sides of the right triangle to write the ratios of the right triangle.



9. $\sin \mathrm{C} =$

$\cos \mathrm{C} =$

$\tan \mathrm{C} =$

10. $\csc \mathrm{C} =$

$\sec \mathrm{C} =$

$\cot \mathrm{C} =$

11. $\sin \mathrm{B} =$

$\cos \mathrm{B} =$

$\tan \mathrm{B} =$

12. $\csc \mathrm{B} =$

$\sec \mathrm{B} =$

$\cot \mathrm{B} =$

For Problem 13-16, given the triangle, solve the problem.

13. Given $\Delta\mathrm{CLD}$ with right angle L, is the measure of $\angle \mathrm{D}\ 67.64°$ if the measure of $\mathrm{DL} = 3.7$ m. and the measure of $\mathrm{CD} = 9.73$ m. (Round to the nearest thousandth.)

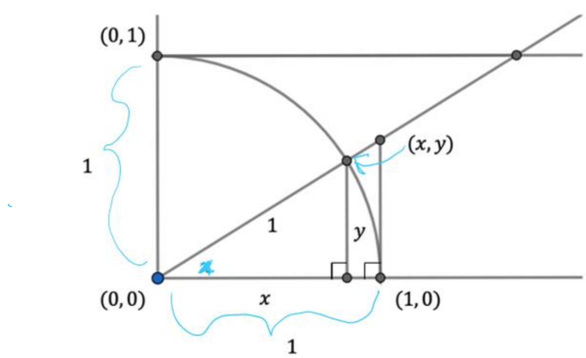
14. Given $\Delta\mathrm{PMO}$ with right angle O, and given the measure of $\mathrm{MP} = 9$ m. and $\cos\angle\mathrm{M}$ is 0.3456, find the measure of MO. (Round to the nearest ten-thousandth.)

15. Given ΔMRS with right angle S, and given the tangent of $\angle R$ is 1.75 and the length of $RS = 6$ cm., find the length of MS. (Round to the nearest thousandth.)

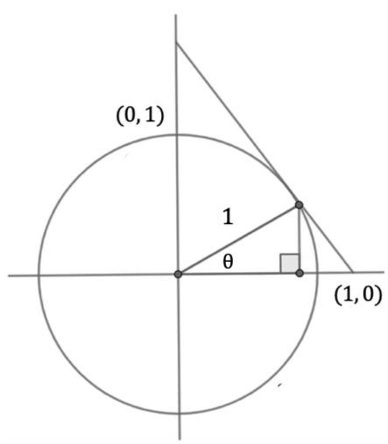
16. What is the length of MR in ΔMRS in Problem 15?

For Problem 17 and 18, label the parts of the diagram as $\sin\theta$, $\cos\theta$, $\tan\theta$, $\sec\theta$, $\csc\theta$, and $\cot\theta$ for the graphical representation of the trigonometric functions or $\sin x$, $\cos x$, $\tan x$, $\sec x$, $\csc x$, and $\cot x$.

17.

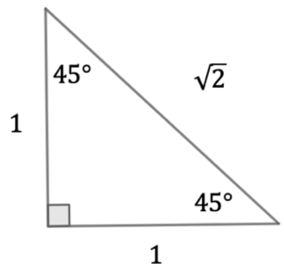


18.



For Problem 19 and 20, use the triangle given to solve the problem.

19. Given the $45° - 45° - 90°$ triangle, find the value of the following functions. Write the value in exact form.

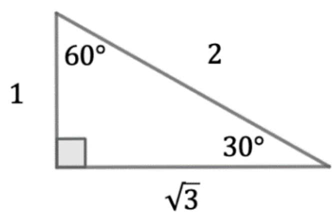


$\sin 45°$ $\csc 45°$

$\cos 45°$ $\sec 45°$

$\tan 45°$ $\cot 45°$

20. Given the $30° - 60° - 90°$ triangle, find the value of the following functions. Write the value in exact form.



$\sin 60°$ $\csc 60°$

$\cos 60°$ $\sec 60°$

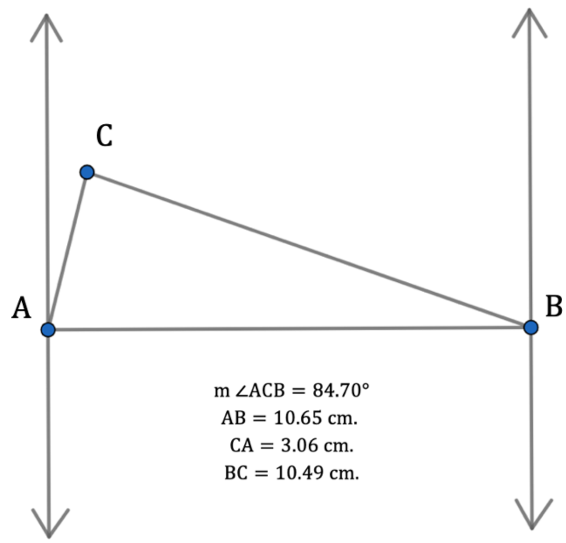
$\tan 60°$ $\cot 60°$

Section 6.13 Oblique Triangles

Practice Problems 6.13

For Problem 1 and 2, use the diagram below to solve the problem.

m ∠ACB	AB	CA	BC
84.70°	10.65 cm.	3.06 cm.	10.49 cm.



Calculate: $\mathrm{AB} \cdot \mathrm{AB} - (\mathrm{CA} \cdot \mathrm{CA} + \mathrm{BC} \cdot \mathrm{BC}) =$ ________________

What kind of number do you get? Write the number using a, b, and c for the sides, and an inequality symbol to show what it is greater than or less than. Now, convert it so it is a form of the Pythagorean Theorem with an inequality symbol.

2. Write the conjecture to Problem 1 as a conditional statement.

For Problem 3-5, given the set of three numbers, use what you have learned in your previous investigations to tell whether the triangles with those side lengths are right, obtuse, or acute.

3. 5, 6, 9

4. 13, 13, 13

5. 4, 7, 7

For Problem 6 and 7, use the sets from Problem 4 and 5 to solve the problem.

6. Name the triangle in Problem 4 by its sides. How does this help you know the type of triangle it is by the angles?

7. What type of triangle is the triangle in Problem 5 when named by its sides? It has the same name when its sides are 5, 5, 9. How do they differ by the angles? Can you make any conjectures as to the relationship between the lengths of the sides of this triangle and the angles of this triangle?

For Problem 8-10, use the information given to solve the problem.

8. We know the Babylonians did much of their mathematics on clay tablets: one incredibly famous tablet from their explorations is called the "Plimpton 322," and is dated between 1900 B.C. and 1600 B.C. On that tablet are many rows of numbers. Each row of numbers in the table is a triple. One side row includes these three numbers: Side 1 (60); Side 2 (45); Side 3 (75).

Determine whether the triangle formed is a right triangle, obtuse triangle, or an acute angle. If you triple these side lengths, does the result show the same relationship?

9. We have already said that three segments do not always form a triangle. Below are three sets of distances of line segments in inches:

$(1, 4, 5)$ $(2, 3, 6)$ $(9, 10, 25)$

Look at the lengths of the shorter sides compared to the longer side in each set. Explain why these cannot be used to form triangles. Write your conjecture as a conditional statement.

10. Which two sets of the three sets of numbers below do not form a triangle? Tell whether the other set is a right, obtuse, or acute triangle.

10, 24, 36

12, 16, 24

7, 10, 19

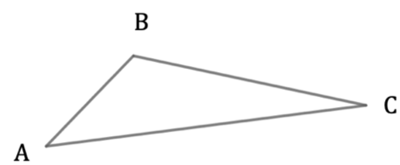
For Problem 11-14, fill in the blanks.

11. If one side of a triangle is longer than another side, then the angle that is opposite the longer side is ____________________ than the angle that is opposite the shorter side.

12. If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side that is opposite the ____________________ angle.

13. The sum of the lengths of any two sides of a triangle is _____________________ than the length of the third side of the triangle.

14. In ΔABC, $\text{AB} + \text{BC} > \text{AC}$, $\text{AB} + \underline{\qquad\quad} > \text{BC}$, $\text{AC} + \text{BC} > \text{AB}$.



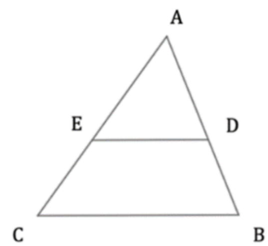
For Problem 15-16, given ΔABC, and given $\beta = 30°$ and the measurement of side $a = 5$, solve the problem. Determine how many triangles are possible given the measurement of side b?

15. a) $b = 1$ b) $b = 3$

16. a) $b = 6$ b) $b = 2.5$

For Problem 16, given the diagram below, solve the problem.

17.



$\angle A \cong \angle A$

$\angle B \cong \angle D$

$\angle C \cong \angle E$

Name the similar triangles in the diagram. If three angles of one triangle are congruent to three angles of another triangle, how many similar triangles are possible?

For Problem 18-20, use the given information to solve the problem.

18. Find the area of ΔABC if $a = 7$, $c = 4$, and $\beta = 50°$. (Round to the nearest hundredth.)

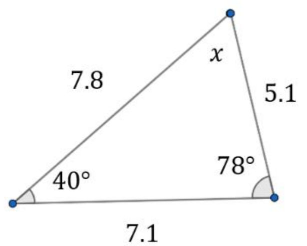
19. What is the semiperimeter (s) of ΔABC with sides $a = 3$, $b = 5$, and $c = 7$?

20. Use Heron's Formula to find the area of ΔABC in Problem 19? (Round to the nearest tenth.)

Section 6.14 Module Review

For Problem 1-4, solve for the angle x in the triangle given and name the triangle by its sides and angles.

1.



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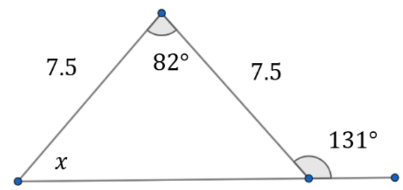

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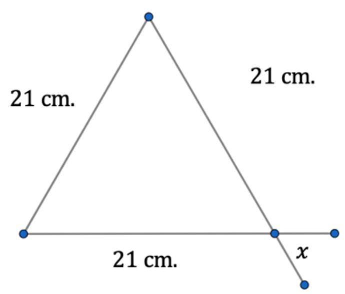

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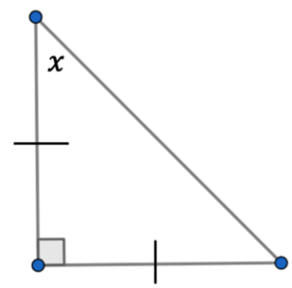
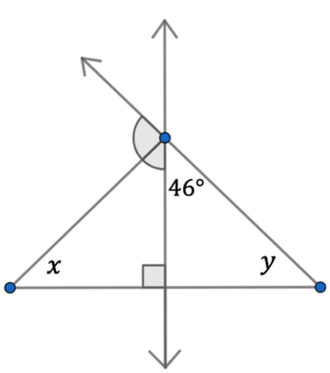


For Problem 5, use the diagram to solve the problem.

5. Find the measure of angles x and y in the triangle if the exterior angle shown is 92°.



For Problem 6-11, tell whether the statement is true or false.

6. If three sides of one triangle are congruent to three sides of another triangle, then the two triangles are congruent to one another.

7. If two sides of one triangle are congruent to two sides of another triangle, then the two triangles are congruent to one another.

8. If two sides and an angle in between the sides of a triangle are congruent to two sides and the angle in between the sides of another triangle, then the two triangles are congruent.

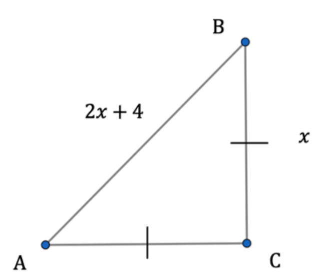
9. All three angles in an isosceles triangle are congruent.

10. If two angles in one triangle are congruent to two angles in another triangle, then the third pair of angles will be congruent.

11. If a triangle has only two congruent angles, then the triangle is isosceles.

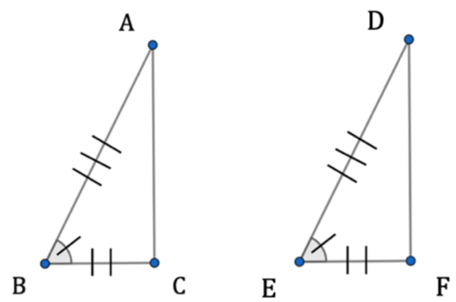
For Problem 12, use the triangle given to solve the problem.

12. If triangle ABC has a perimeter of 108 mm., what is the measure of each side?



For Problem 13-15, if possible, state the congruency theorem that makes triangle ABC congruent to triangle DEF.

13.



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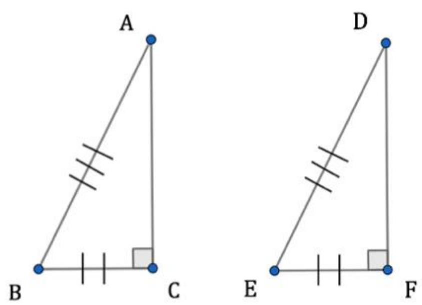

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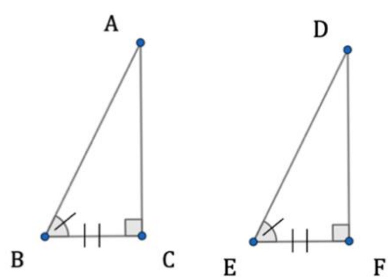
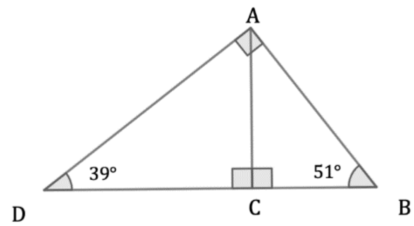


For Problem 16-20, use the triangle below to solve the problem. Let triangle DAC be a scalene triangle.



16. Find the measure of angle DAC.

17. Find the measure of angle BAC.

18. Is triangle DAC congruent to triangle ABC?

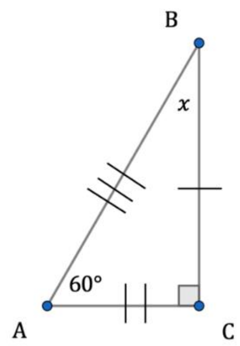
19. What type of triangles are DAC, ABC, and BDA?

20. Is triangle DAC similar to triangle ABC? (To be similar the corresponding sides must be proportional, and the corresponding angles must be congruent.)

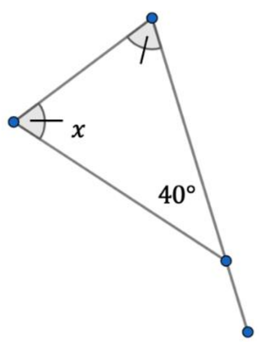
Section 6.15 Module Test

For Problem 1-3, solve for angle x in the triangle given and name the triangle by its sides and angles.

1.



2.



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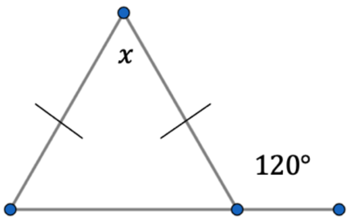
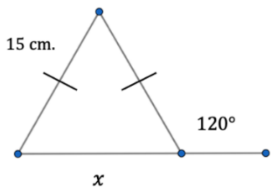


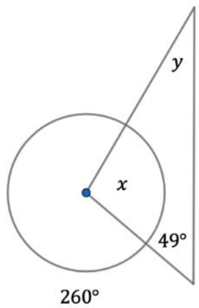
For Problem 4, solve for side length x in the triangle given and name the triangle by its sides and angles.

4.



For Problem 5, use the diagram to solve the problem.

5. Find the measure of the missing angles x and y. The third angle is 49°.



For Problem 6-11, tell whether the statement is true or false.

6. If an angle and a side of one triangle are congruent to an angle and a side of another triangle, then the two triangles are congruent to one another.

7. If one side of a triangle is congruent to one side of another triangle, then the other two pairs of sides will also be congruent.

8. If two angles and a side in between the angles of a triangle are congruent to two angles and the side in between the angles of another triangle, then the two triangles are congruent.

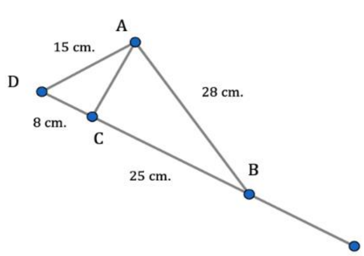
9. All three sides in an acute triangle are congruent.

10. If three angles in one triangle are congruent to three angles in another triangle, then the two triangles will be congruent.

11. The sum of the lengths of any two sides of a triangle is less than the length of the third side.

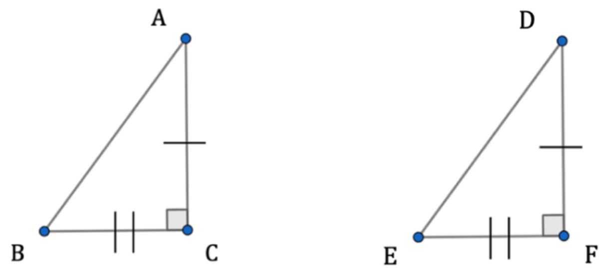
For Problem 12, use the triangle given to solve the problem.

12. If triangle ABC has a perimeter of 76 cm., what is the perimeter of triangle ACD?



For Problem 13-15, state the congruency theorem that makes triangle ABC congruent to triangle DEF.

13.



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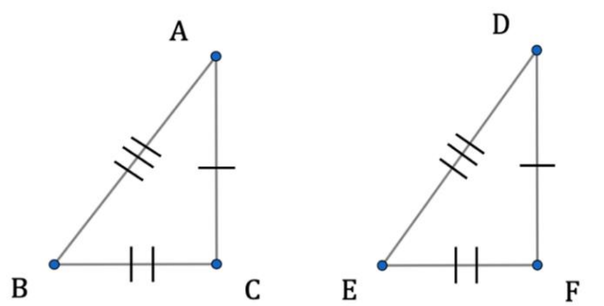

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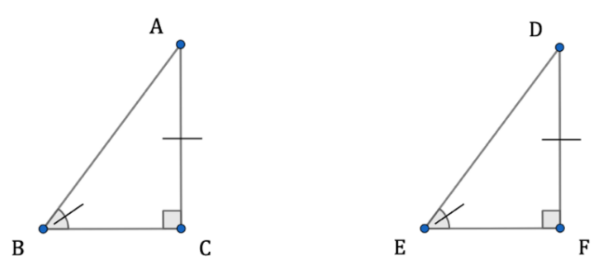
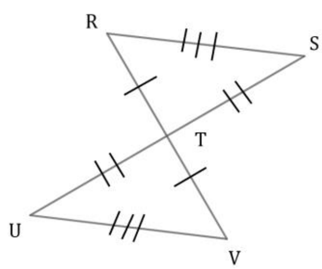


For Problem 16-19, fill in the blanks.

16. In an isosceles triangle, the angle bisector between the two congruent sides is also the ____________________ and the ____________________ of the triangle.

17. If the shortest distance between two points is a _____________________ line, then in triangle DEF, the distance between D and E is the line _____________________ connecting them, and the distance from D to F and F to __________ must be longer than the distance between D and E. This is the _____________________ _____________________ _____________________.

18.



$\angle RTS \cong \angle VTU$ by the _____________________ _____________________ _____________________ _____________________

$\Delta RTS \cong \Delta VTU$ by the __________-___________-__________ _____________________ _____________________

19. If an exterior angle is double the measure of one remote interior angle, and the angles are not all congruent, then the triangle is _____________________.

For Problem 20, match the definition with its point of concurrency.

I. The point where the perpendicular bisectors of the three sides of a triangle meet. a) Orthocenter

II. The point where the three angle bisectors of a triangle meet. b) Incenter

III. The point where the three medians of a triangle meet. c) Circumcenter

IV. The point where the three altitudes or the lines containing the altitudes meet. d) Centroid