Algebra 2 Module 8 Exponents and Logarithms

Section 8.1 Introducing Exponential Functions

Practice Problems 8.1

To begin you will do the following experiment on your own:

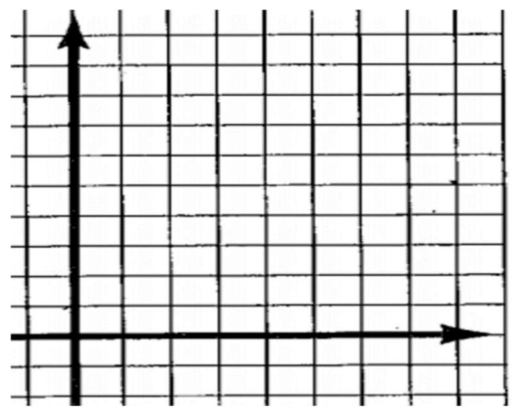
1. Take a piece of tape or chalk outside along with some kind of ball (basketball, tennis ball, etc.)
2. Drop the ball along a brick wall from a starting height of 200 centimeters, then put a chalk mark or piece of tape on the wall to where it rebounds to after you drop it.
3. Do three trials at a drop of 200 cm. and then add them together and divide by three to get the average rebound height. Calculate the rebound ratio by dividing the average rebound height by the drop height.
4. Repeat step 3 but with drops from 175 cm., 150 cm., and 125 cm., and 100 cm., and complete the table below.
5. Find the average rebound ratio. This is the constant of variation. If x is the starting height and y is the rebound height, what is the equation that models the experiment?

Start Height (cm.)	**1st Trial**	**2nd Trial**	**3rd Trial**	**Avg. Rebound Height (cm.)** (mean of the measurements of the first three trials)	**Rebound Ratio**
200					
175					
150					
125					
100					

Now you will perform a second experiment using the same ball from the first experiment.

1. Drop a ball six times from 200 cm. each time.
2. After the first drop, have someone catch the ball after the first bounce and measure the bounce height.
3. After the second drop, have someone catch the ball after the second bounce and measure the bounce height of the second bounce.
4. After the third drop, have someone catch the ball after the third bounce and measure the bounce height of the third bounce.
5. After the fourth drop, have someone catch the ball after the fourth bounce and measure the bounce height of the fourth bounce.
6. After the fifth drop, have someone catch the ball after the fifth bounce and measure the bounce height of the fifth bounce.
7. After the sixth drop, have someone catch the ball after the sixth bounce and measure the bounce height of the sixth bounce.
8. If x is the bounce number and y is the bounce height, draw the graph of the results.
9. Use the rebound ratio calculated in the first experiment to find the exponential equation that models this experiment.
10. Use the equation to calculate the exact value of each bounce height. How closely does this relate to the experimental data?

Bounce Number	**Bounce Height**



Below are results from the experiment performed by Tricia, Kala, and Ciré. Complete the final column of the table.

Start Height (cm.)	**1st Trial**	**2nd Trial**	**3rd Trial**	**Average Rebound Height (cm.)** (mean of the measurements from the three trials)	**Rebound Ratio**
200	92	91	98	93.6	
150	81	80.5	85.5	82.3	
100	60.5	60	60	60.1	
50	31	27	30.5	29.5	
25	13	11	9.5	11.2	

1. Calculate the average rebound ratio.
2. Use the general formula for exponential equations ($y = ab^x$) to find the predicted rebound heights from the five different start heights. Let a be the start height, let b be the average rebound ratio, and let $x = 1$ (for one bounce). Compare the predicted rebound heights to the actual rebound heights in the table above. The closer the results the more reliable the ball is according to the manufacturer's rebound ratio specifications.
3. How does the exponential equation compare to the general formula for linear equations ($y = kx$) where k is the constant of variation?

Section 8.2 Graphs of Exponential Functions
Practice Problems 8.2

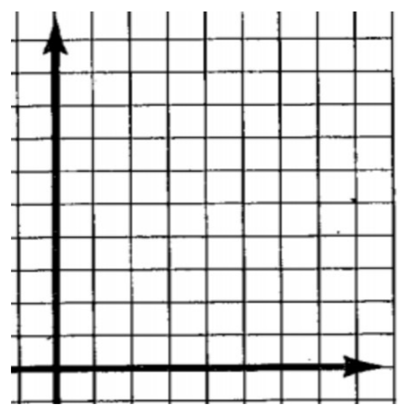
1. Assume the sequence is arithmetic: 3, 6, …
   a. Write the next four terms:

   b. What is being added each time? The constant rate of increase or decrease is called the common difference d.

   c. What is the first term of the sequence? The first term in an arithmetic sequence is a_1 and the nth term of a sequence is a_n.

   d. Complete the table and sketch a graph of the discrete data.

Term Number (n)	**Term Value (a_n)**
1	3
2	6
3	9
4	12
5	15
6	18



2. The recursive formula for an arithmetic sequence is $a_n = a_{n-1} + d$ for $n \geq 2$ and n is an integer. Write the recursive formula for the arithmetic sequence. If n is the nth term, then $n - 1$ is the term before it.

3. Use the recursive formula to find the 7th term of the arithmetic sequence.

4. Why is $n \geq 2$?

Use the sequence 18, 14, 10, 6, … to solve Problem 5 – 9.

5. What is the common difference for the arithmetic sequence?

6. Write the recursive formula for the arithmetic sequence.

7. Use the recursive formula to find the 5th term of the arithmetic sequence.

8. The explicit formula for the nth term of an arithmetic sequence is $a_n = a_1 + (n-1)d$, where n is the term number and a_1 is the first term of the sequence. Write the explicit formula for the arithmetic sequence.

9. Use the explicit formula to find the 50th term of the arithmetic sequence.

10. Write a recursive formula for $a_n = 8n + 2$.

11. Assume the sequence is geometric: 3, 6, …

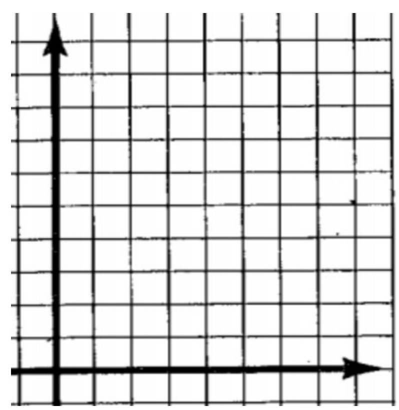
    a. Write the next four terms:

    b. What is being multiplied each time? This is called the common ratio r.

    c. What is the first term of the sequence? The first term in a geometric sequence is g_1 and the nth term of a sequence is g_n.

    d. Complete the table and sketch a graph of the discrete data.

Term Number (n)	**Term Value (g_n)**
1	3
2	6
3	12
4	24
5	48
6	96



12. The recursive formula for a geometric sequence is $g_n = r \cdot g_{n-1}$ where r is the common ratio. Write the recursive formula for the geometric sequence.

13. Use the recursive formula to find the 7th term of the geometric sequence.

Use the geometric sequence 2, -8, 32, -128, … to solve Problem 14 – 17.

14. What are the next three terms of the sequence.

15. What is the common ratio and how do you know it is negative?

16. The explicit formula for a geometric sequence is $g_n = g_1 r^{n-1}$ where g_1 is the first term, r is the common ratio ($r \neq 0$), and n is any integer. What is the explicit formula for the geometric sequence?

17. Use the explicit formula to find the 10^{th} term of the geometric sequence.

18. Write an explicit formula for $g_n = 0.2g_{n-1}$ for $n \geq 2$ when $g_1 = 11$.

19. Write a recursive formula for $g_n = 3 \cdot (\frac{1}{2})^{n-1}$.

20. The first swing out of a pendulum is 28 feet for the length of the arc. The swing back is only 70% of that length and the following swing out is only 70% the length of the arc from the previous swing back. What will be the length of the arc for the fourth swing, which is back only?

Section 8.3 Transformations of Exponential Functions

Practice Problems 8.3

For Problem 1, use the information below to create the graph.

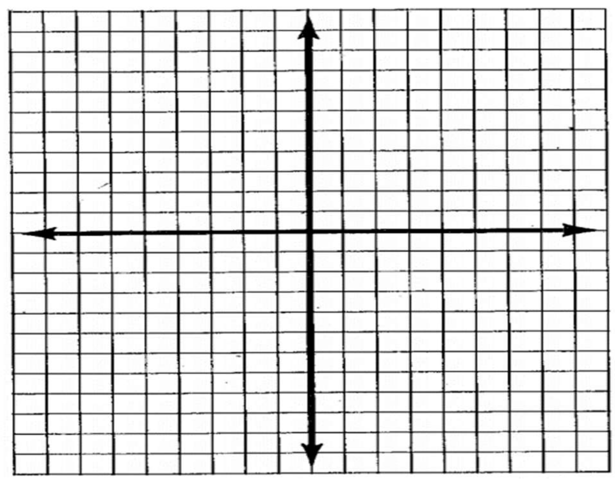
The standard form of an exponential function is $y = ab^x$. Substituting values in for x and solving for y will allow you to create a table and then graph the ordered pairs.

Standard form can be converted to graphing form $ab^{(x-h)} + k = y$ to graph the equation more readily.

The parent function for a quadratic equation is $y = b^x$ where $b > 0$ but $b \neq 1$.

1. Given the table for the exponential function $y = b^x$ where $b = 2$, draw the graph.

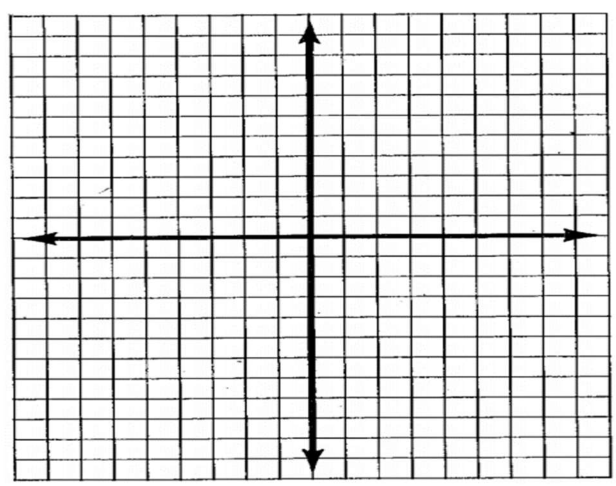
$\boldsymbol{x}$	$\boldsymbol{y}$ or $\boldsymbol{f(x)}$
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8



For Problem 2-4, investigate the parameter a in exponential functions.

2. Let $a > 1$ and make a table and graph for $y = 4 \cdot 2^x$.

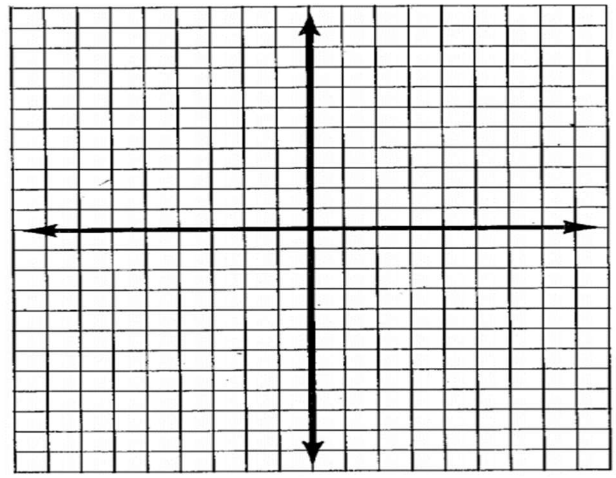
$\boldsymbol{x}$	$\boldsymbol{y}$ or $\boldsymbol{f(x)}$
-3	
-2	
-1	
0	
1	
2	
3	



Name the parameter you checked and describe the effect it had on the parent function to cause the transformation.

3. Let $0 < a < 1$ and make a table and graph for $y = \frac{1}{3} \cdot 2^x$.

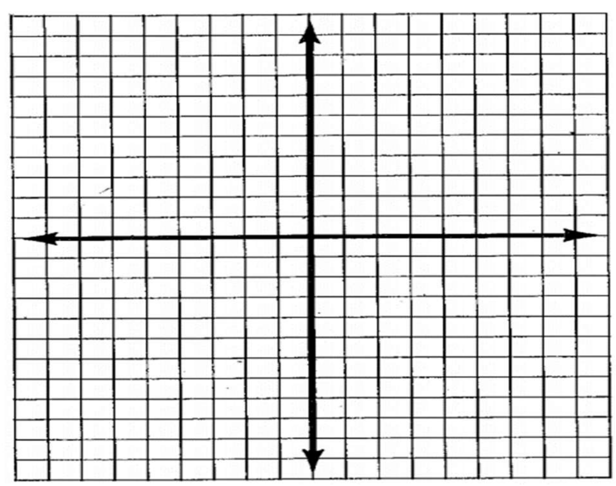
x	y or $f(x)$
-3	
-2	
-1	
0	
1	
2	
3	



Name the parameter you checked and describe the effect it had on the parent function to cause the transformation.

4. Let $a < 0$ and make a table and graph for $y = -4 \cdot 2^x$.

x	y or $f(x)$
-3	
-2	
-1	
0	
1	
2	
3	

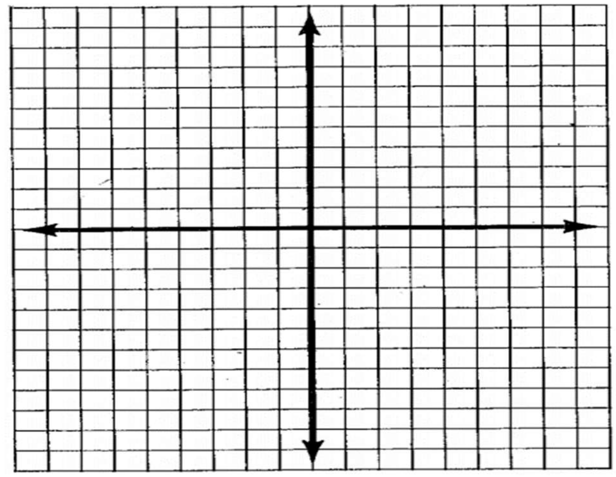


Name the parameter you checked and describe the effect it had on the parent function to cause the transformation.

For Problem 5-6, investigate parameter h for exponential functions.

5. Let h be a positive number and make a table and graph for $y = 3^{(x-2)}$.

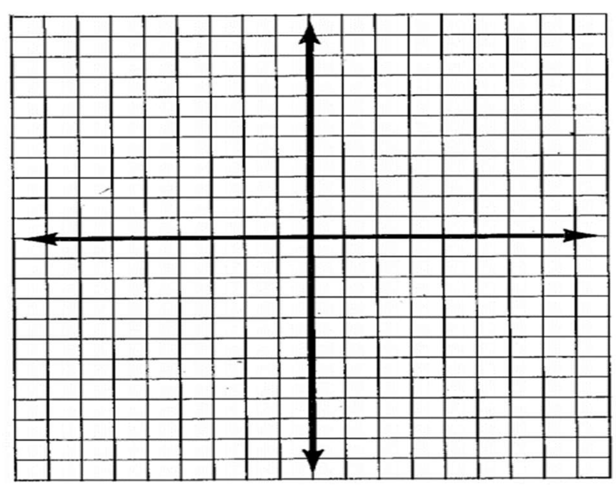
$\boldsymbol{x}$	$\boldsymbol{y}$ or $\boldsymbol{f(x)}$
-1	
0	
1	
2	
3	
4	
5	



Name the parameter you checked and describe the effect it had on the parent function to cause the transformation.

6. Let h be a negative number and make a table and graph for $y = 3^{(x+2)}$.

$\boldsymbol{x}$	$\boldsymbol{y}$ or $\boldsymbol{f(x)}$
-5	
-4	
-3	
-2	
0	
1	

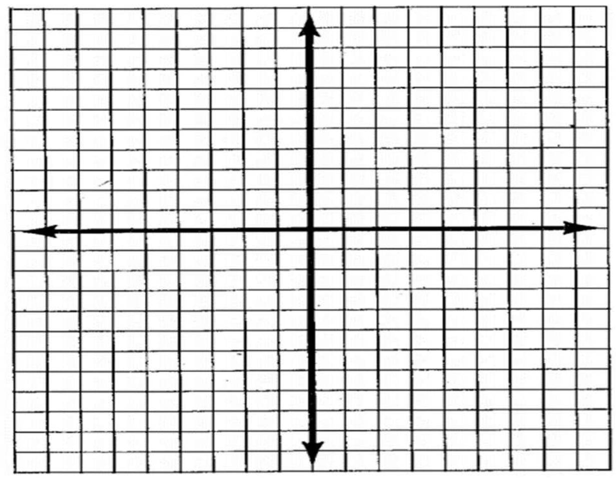


Name the parameter you checked and describe the effect it had on the parent function to cause the transformation.

For Problem 7 and 8, investigate the parameter k for exponential functions.

7. Let $k > 0$ and make a table and graph for $y = 3^x + 5$.

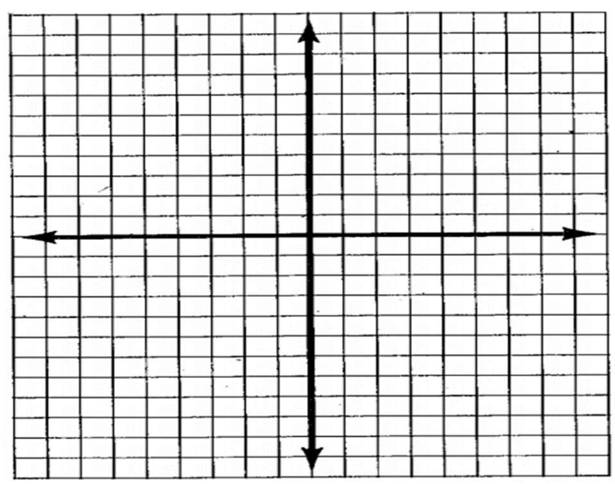
x	y or $f(x)$
-3	
-2	
-1	
0	
1	
2	
3	



Name the parameter you checked and describe the effect it had on the parent function to cause the transformation.

8. Let $k < 0$ and make a table and graph for $y = 3^x - 5$.

x	y or $f(x)$
-3	
-2	
-1	
0	
1	
2	
3	



Name the parameter you checked and describe the effect it had on the parent function to cause the transformation.

Section 8.4 Comparing Exponential Functions and Power Functions

Practice Problems 8.4

In the lesson, we investigated exponential functions and power functions where the base of the exponential function (b) was equivalent to the exponent of the power function (n). Now let's investigate exponential and power functions where $n > b$ and see what happens in the long run.

1. Let $b = 2$ and $n = 3$. Write the exponential and power functions when $a = 1$. Use integer solutions to write the domain for the given questions.

   a. On what interval(s) is $2^x < x^3$? How do you know? Do you need the graph to determine this?

   b. At what point(s) is $2^x = x^3$?

   c. On what interval(s) is $2^x > x^3$? How do you know? Do you need the graph to determine this?

   d. In the long run, is 2^x or x^3 greater for larger values of x?

2. Let $b = 3$ and $n = 5$. Write the exponential and power functions when $a = 1$.

   a. On what interval(s) is $3^x < x^5$? How do you know? Do you need the graph to determine this?

   b. At what point(s) is $3^x = x^5$?

   c. On what interval(s) is $3^x > x^5$? How do you know? Do you need the graph to determine this?

   d. In the long run, is 3^x or x^5 greater for larger values of x?

3. Let $b = 0.5$ and $n = 5$. Write the exponential and power functions when $a = 1$.

a. On what interval(s) is $0.5^x < x^5$? How do you know? Do you need the graph to determine this?

b. At what point(s) is $0.5^x = x^5$?

c. On what interval(s) is $0.5^x > x^5$? How do you know? Do you need the graph to determine this?

d. In the long run, is 0.5^x or x^5 greater for larger values of x?

4. Just for fun, let $b = 5$ (a large base) and $n = 0.5$ (a small base). Write the exponential and power functions when $a = 1$.

a. Which do you think will be greater in the long run?

b. What is the point(s) of intersection when $5^x = x^{0.5}$?

c. What do you notice about the graphs?

5. Complete the table for the given values of x.

x	$y = 4^x$	$y = x^6$	$y = x^{12}$
10			
100			
1,000			

6. a. For what values of x in the table is $4^x < x^6$?

b. For what values of x in the table is $4^x > x^6$?

c. In the long run, is 4^x or x^6 greater for larger values of x?

7. a. For what values of x in the table is $4^x < x^{12}$?

b. For what values of x in the table is $4^x > x^{12}$?

c. In the long run, is 4^x or x^{12} greater for larger values of x?

8. Write a conjecture comparing exponential functions to power functions for larger values of x. Be sure to state which will be larger in the long run and why.

Section 8.5 Problem Solving Using Exponential Equations

Practice Problems 8.5

For Problem 1-6, use the equation $A = \$2{,}075(1 + 0.025)^2$ to answer the questions for a period of one year.

1. What is the initial deposit made in the account?

2. How many times in the year was the principal compounded if it had an annual interest rate?

3. What is the annual interest rate?

4. How much is in the account at the end of the year?

5. How many times will the interest be compounded in three years?

6. How much money will be in the account at the end of three years?

For Problem 7-10, use the following scenario to answer the questions:

Bessie was saving for a down payment for a house. She put $8,000.00 in an account for four years at an annual interest rate of 4% compounded bi-annually (twice a year). Bessie needs to have at least $10,000.00 to make the down payment.

7. What is the principal (P) in the scenario?

8. How many times was it compounded in four years?

9. What is the amount Bessie will have after four years? Is it enough to make the down payment?

10. Will Bessie have enough after five years of saving to make the down payment? When will Bessie have enough to make a $10,000.00 down payment?

For Problem 11-20, solve the word problem.

11. You have just inherited \$5,000 from an old, incredibly rich (and smart) math teacher. However, there is one condition: you must decide whether to put the money in an account with a 9% annual percentage rate compounded yearly or an account with an 8% annual rate compounded quarterly. Each account will be compounded for 5 years. If you choose the account that generates the most interest, you will get the money in 5 years. Where do you put the money? Show your work below.

12. You have just been notified that you are the only living descendent of a math-loving eccentric (whose last name is probably not Brown) who put the equivalent of \$10 in a bank account in 1755 and left it. Since then, the money has been collecting interest. He left instructions in his will that in the year 2010, if he had any living descendent who could determine the exact amount of money in the bank, that person could have the money. If not, the account was to be turned over to charity. Can you claim the money? Here is the information you need to figure it out:

    The equivalent of \$10 was deposited January 1, 1755, in a bank account that has an annual interest rate of 5%. Since then, the interest has been compounded annually. If you are correct in calculating the balance as of December 31, 2010, you can have the money. On that day, how much money will be in the account? Show all your work clearly.

13. The number of people that smoke cigarettes in Amador County has been decreasing at a rate of 5% per year. As of September 1994, the number of people who smoke in the county was 4,000. Solve a. and b. below assuming this rate of decrease will continue.
    a. Write an equation that will represent the number of smokers in Amador County in the nth year after 1994.
    b. According to your model, how many smokers will there be in Amador County in September of 2010?

14. Tomas' grandfather put \$1,000 in the bank for him when he was born. The account has been earning 2.5% interest compounded annually. Tomas is now 18 years old and wants to take out the money so he can go to college. How much money does he have now? Show how you got your answer.

15. An account is earning 5% interest, compounded quarterly. How much will it be worth in 8 years if an initial investment of \$5,000 is put into the account and no withdrawals are made?

16. If you have a gold wedding ring that was appraised at $1,800.00 in 1980 and gold increases by 2% a year, what was the value of the ring in the year 2000?

17. Tell whether each of the following exponential functions models growth or decay:

    a. $y = \frac{1}{2}(5)^x$

    b. $y = 0.3(2)^x$

    c. $y = 3(1.1)^x$

    d. $y = 5(0.96)^x$

For Problem 18-20, use the equation $y = 100(\frac{2}{3})^x$ to solve the problem.

18. What is the initial value?

19. What is the growth factor?

20. Is it increasing or decreasing?

Section 8.6 Solving the Parts of an Exponential Equation

Practice Problems 8.6

For Problem 1-4, use the equation $f(x) = \frac{1}{2}(4)^x$ to answer the questions about bacterial growth.

1. What is the initial population?

2. What is the growth factor?

3. The time period for bacterial growth is recorded daily. How many bacteria will there be after a week?

4. After how many days will there be approximately 131,000 bacteria?

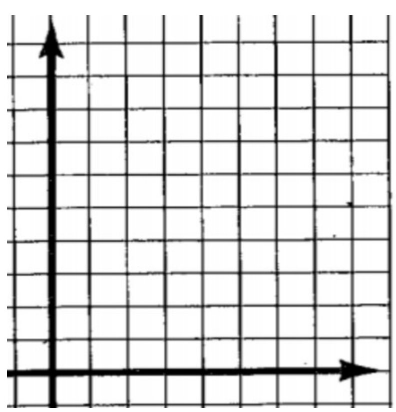
For Problem 5-12, use the following sequence to answer the questions:

$200, 100, 50 \ldots$

5. Is the sequence arithmetic or geometric?

6. List the next three terms of the sequence.

7. Draw the graph with values of x from 1-6. Let x be term number and y be the term value.



8. Is the sequence increasing or decreasing?

9. What is the initial value (start value)?

10. What is the multiplier (base) of the exponential equation?

11. Will the sequence ever include negative numbers? Is the graph discrete or continuous?

12. Will the sequence ever reach zero? What is the horizontal asymptote?

For Problem 13-15, use the following scenario to answer the questions:

A virus is spreading throughout a company. Each day a given number of employees gets the infection.

Day	**Employees who are not Infected**
0	381
1	114
2	34
3	
4	
…	

13. How many employees are in the company?

14. What is the percentage of employees getting infected each day?

15. At what day of the week will all the employees be infected?

For Problem 16-20, use the equation $y = 4(2)^x$, which represents the growth of a company started by a young entrepreneur.

16. How many employees did the young entrepreneur have when he first started the company?

17. How many employees will the young entrepreneur have after 5 years if x represents years?

18. What would the equation be if the young entrepreneur started the company with three employees?

19. The number of employees working for the company is doubling each year. What percentage of growth is this?

20. When will the company have approximately 4,000 employees?

Section 8.7 Inverses of Exponential Functions

Practice Problems 8.7

For Problem 1-10, use the equation $y = 3^x$ to answer the questions.

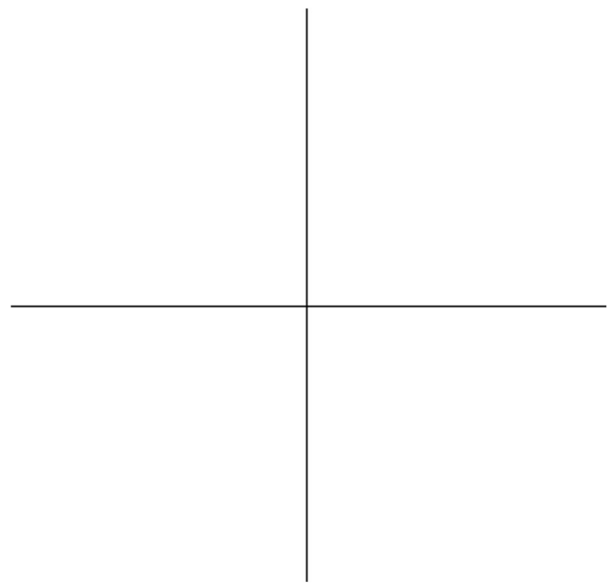
1. What is the value of a?

2. What is the value of b?

3. Complete the table for the equation.

x	y
-6	
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	
6	

4. Sketch the graph of the exponential equation.



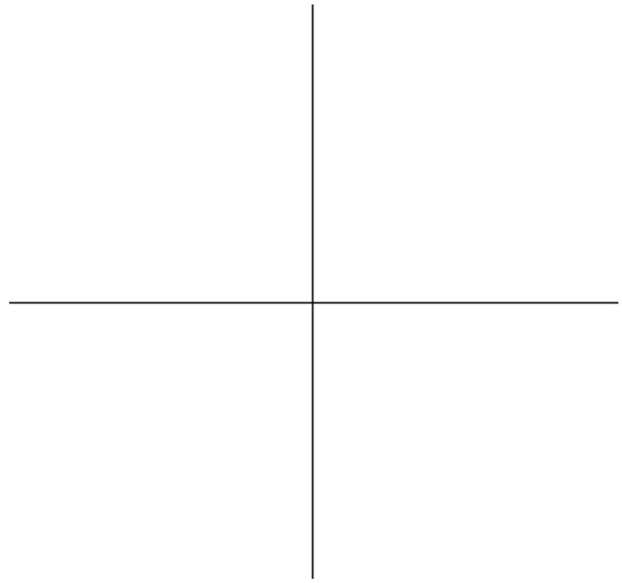
5. What is the y-intercept of the equation?

6. What is the equation for the horizontal asymptote?

7. Complete the table for the inverse of the equation.

x	y
	-6
	-5
	-4
	-3
	-2
	-1
	0
	1
	2
	3
	4
	5
	6

8. Sketch the graph that is of the inverse of the equation.



9. Write the equation as a logarithm.

10. Write the inverse of the equation as a logarithm.

For Problem 11-15, write the exponential equation and it's equivalent in logarithmic form.

11. $y = 5^x$

12. $y = 0.34^x$

13. $y = (\frac{1}{3})^x$

14. $y = 7^x$

15. $y = 4.4^x$

For Problem 16-20, find the inverse of the exponential equation. Write it in logarithmic form.

16. $y = 6^x$

17. $y = 0.1^x$

18. $y = 54^x$

19. $y = (\frac{2}{7})^x$

20. $y = 4.4^x$

Section 8.8 Converting Exponential and Logarithmic Equations

Practice Problems 8.8

For Problem 1-10, write the exponential equation in logarithmic form.

1. $2^3 = 8$

2. $2^{-3} = \frac{1}{8}$

3. $9^{\frac{3}{2}} = 27$

4. $10^{-2} = 0.01$

5. $3^{-4} = \frac{1}{81}$

6. $4^2 = 16$

7. $7^2 = 49$

8. $10^{-2} = \frac{1}{100}$

9. $5^0 = 1$

10. $16^{\frac{1}{2}} = 4$

For Problem 11-19, write the logarithmic equation in exponential form.

11. $\log_2(\frac{1}{8}) = -3$

12. $\log_3 81 = 4$

13. $\log_9 3 = \frac{1}{2}$

14. $\log_{10} 100 = 2$

15. $\log_3 9 = 2$

16. $\log_7 \sqrt{7} = \frac{1}{2}$

17. $\log_5(\frac{1}{125}) = -3$

18. $\log_3 27 = 3$

19. $\log_3(\frac{1}{81}) = -4$

For Problem 20, solve the word problem.

20. What two consecutive integers does $\log_{10} 101$ lie between?

Section 8.9 Solving for the Variable in Logarithmic and Exponential Equations

Practice Problems 8.9

For Problem 1-10, write each logarithm in exponential form and then solve for x.

1. $\log_x 16 = 4$

2. $\log_x 16 = 2$

3. $\log_x 9 = 2$

4. $\log_x 27 = 3$

5. $\log_{\sqrt{16}} x = \frac{1}{2}$

6. $\log_2 x = 4$

7. $\log_6 x = \frac{1}{6}$

8. $\log_5 x = 2$

9. $\log_{10} x = 1$

10. $\log_{10} x = 0$

11. $\log_6 6 = x$

12. $\log_5 5 = x$

13. $\log_2 2\sqrt{2} = x$

14. $\log_4 16 = x$

15. $\log_3 1 = x$

16. $\log_2(\frac{1}{2}) = x$

17. $\log_5(\frac{1}{5}) = x$

18. $\log_{16} \sqrt{2} = x$

19. $\log_3 81 = x$

20. $\log_3 \sqrt[3]{3} = x$

Section 8.10 Change of Base and Logarithm of a Power

Practice Problems 8.10

For Problem 1-3, use the logarithm $\log_a b = x$ to solve the problems.

1. Evaluate $\log_a 1$.

2. Evaluate $\log_a a$.

3. Evaluate $\log_a a^x$.

For Problem 4-10, use the change of base formula to evaluate the problem.

$$\log_a b = \frac{\log_{10} b}{\log_{10} a}$$

4. $\log_6 2$

5. $\text{Log}_{-1} 4$

6. $\log_4(-1)$

7. $\log_5 3$

8. $\log_3 9$

9. $\log_{0.7} 7$

10. $\log_{\frac{1}{2}} 8$

For Problem 11-15, use "The Log of a Power Property" to evaluate the expression.

11. $\log_3 3^2$

12. $\log_{\frac{1}{2}} 4^3$

13. $\log_{0.25} 2^5$

14. $\log_2 3^{\frac{1}{2}}$

15. $\log_3 2^{0.75}$

For Problem 16-20, solve the word problem.

16. Which is greater, $2 \log_3 4$ or $\log_4 3^2$?

17. Which is smaller, $f(x) = \log_2 4^x$ when $x = \frac{1}{4}$ or $f(x) = \log_x 243$ when $x = 3$.

18. If $\log_5 x^2 = \log_5(3x - 10)^2$ then you can divide by $\log 5$ on both sides of the equation and solve for x.

19. If $\log_9(2x - 10) = \log_9(x^2 - 5x)$, then solve for x. Firstly, divide both sides of the equation by $\log_9$.

20. The equation $93 \log d + 65 = s$ models the travel of a tornado. The speed of the wind near the center of the tornado (s) is measured in miles per hour and is related to the distance (d) it travels in miles. If a tornado travels 200 miles in less than thirty minutes, what is the speed of the wind near the center of the tornado?

Section 8.11 Operations and Properties of Logarithms

Practice Problems 8.11

For Problem 1-4, expand the logarithmic expression using the Properties of Logarithms.

1. $\log_5(\frac{x}{y})$

2. $\log_2 5x^3$

3. $\log_{10}(\frac{0.6}{5})$

4. $\log_3(2 \cdot 6)$

For Problem 5-8, compress the expanded logarithm using the Properties of Logarithms if possible.

5. $\log_3 5 - \log_3 8$

6. $\log_3 5 + \log_3 4$

7. $\log_4 2 + \log_2 8$

8. $\log_{10} 10 - \log_{10} 100$

For Problem 9-12, use the following information to solve:

The magnitude of an earthquake is measured using the Richter Scale where R represents the magnitude. The formula $R = \log_{10} I$ gives the intensity, I, of the measure of the wave energy of an earthquake per unit of area.

9. Use the formula above for an earthquake that is a 6.9 on the Richter Scale, $R = 6.9$. Rewrite it as an exponential equation to solve for I, intensity.

10. Use the formula above for an earthquake that is a 7.5 on the Richter Scale, $R = 7.5$. Rewrite it as an exponential equation to solve for I, intensity.

11. What is the increase in magnitude on the Richter Scale from a 6.9 earthquake to a 7.5?

12. What is the scale factor of increase for the intensity change from Problem 11? In other words, how many times greater was the 7.5 earthquake than the 6.9 earthquake?

For Problem 13-20, complete the Logarithm Table below.

		Product Property	**Product Property**	**Quotient Property**	**Quotient Property**
13.	$\log_2 20$	$\log_2 4 +$ ______	$\log_2 10 + \log_2 2$	$\log_2 40 -$ ______	$\log_2 100 - \log_2 5$
14.	$\log_4 24$	$\log_4 6 +$ ______	$\log_4 8 +$ ______	$\log_4 72 - \log_4 3$	$\log_4 120 - \log_4 5$
15.		$\log_5 2 + \log_5 8$	$\log_5 4 + \log_5 4$	$\log_5 32 - \log_5 2$	$\log_5 64 - \log_5 4$
16.	$\log_3 30$	$\log_3 2 +$ ______	$\log_3 3 +$ ______	$\log_3 90 -$ ______	$\log_3 120 -$ _____
17.	$\log_{10} 100$	$\log_{10} 50 + \log_{10} 2$	$\log_{10} 4 + \log_{10} 25$	$\log_{10} 1{,}000 -$ ______	$\log_{10} 300 -$ _____
18.	$\log_6 45$	$\log_6 9 +$ ______	$\log_6 3 +$ ______	$\log_6 90 -$ ______	$\log_6 225 -$ _____
19.					
20.					

Section 8.12 Applications of Logarithms

Practice Problems 8.12

For Problem 1-5, fill in the table and then use it to solve the problem.

1. As previously stated in this module, any number may be written as a power of 10. Complete the table below for $y = 10^x$ (remember, $y = 10^x$ means $\log_{10} y = x$).

x	10^x
0	1
	2
	3
	4
	5
	6
	7
	8
	9
1	10

2. Draw the graph of x on the x-axis and draw the graph of 10^x on the y-axis. What type of equation models this graph?

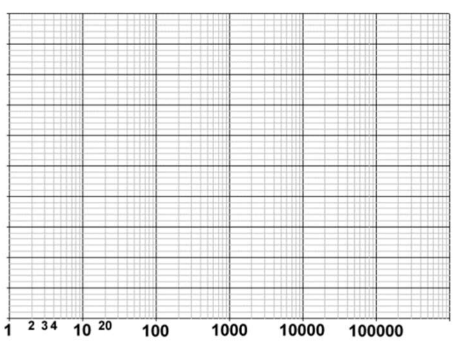
3. Draw the graph with 10^x on the x-axis and the x values on the y-axis. What type of equation models this graph?

4. The numbers for 10^x are the logarithmic scale for the numbers x on the linear scale. Write the numbers for the y-scale (logarithmic) above where they are located on the linear scale given below, which goes from 0 to 1.

5. What do you notice about the numbers on the logarithmic scale?

For Problem 6 and 7, use the explanation given to solve the problem.

6. Semi-log graphs have the logarithmic scale on one axis and the linear scale on the other axis. If the exponential function $y = 10^x$ was graphed on semi-log paper, what type of function would it appear to be?



7. Log-log paper is used when both the horizontal axis and vertical axis are logarithmic scales. If the exponential function $y = 10^x$ was graphed log-log paper, what type of function would it appear to be?

For Problem 8-10, use the information given to solve the problem.

8. As stated in the lesson notes, the pH scale goes from 0 to 14.

a) Find $[H^+]$ (the concentration of hydrogen ions) in the table below. Use the formula from Example 3 in the lesson notes.

pH	**Concentration of Hydrogen Ions**
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	

b) Graph the data from Problem 8 with pH on the x-axis and hydrogen ions on the y-axis. What is difficult about this scale?

9. a) Find the logarithmic scale for the concentration of hydrogen ions for a pH of 1-14 in the table below.

pH	$\mathbf{log_{10}[H]^+}$
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	

b) Graph the data with pH on the x-axis and with the logarithm of the hydrogen ions on the y-axis. Why is the scale easier to read?

10. Why do you think a logarithmic scale is used? This process is called linearizing the data. How does it help?

Section 8.13 The Natural Logarithm and its Inverse

Practice Problems 8.13

For Problem 1-3, solve the expression.

1. What is $\ln(e)$?

2. What is $\ln(0)$?

3. What is $\ln(-3)$?

For Problem 4-8, expand each expression.

4. $\ln(\frac{8}{6})$

5. $\ln(4x^2)$

6. $\ln(\frac{1}{2})$

7. $\ln(\frac{y^2}{x^2})$

8. $\ln(10xy)$

For Problem 9-12, compress each expression.

9. $\ln 2 - \ln 5$

10. $\ln 3 + \ln 7$

11. $5 \ln x + 3 \ln y$

12. $\ln \frac{1}{2} + \ln x + \ln y$

For Problem 13-15, use the Rule of 72 to solve the problem.

13. How long does it take an investment to double at 12% growth?

14. How long does it take an investment to double at 9% growth?

15. How long does it take an investment to double at 8% growth?

The exponential equation $2^x = 16$ can be rewritten $2^x = 2^4$ and $x = 4$. The equation $2^x = 11$ can be rewritten $\log_2 2^x = \log_2 7$. Since $\log_2 2^x = x$, then $x = \log_2 7$ and using the calculator, $\log_2 7 = 2.80735$.

For Problem 16 and 17, solve for *x* in the exponential equations.

16. $3^x = 10$

17. $4^x = 20$

The exponential equation $e^x = 70$ can be rewritten $\ln e^x = \ln 70$. Since $\ln e^x = x$, then $x = \ln 70$, and using the calculator, $\ln 70 \approx 4.2485$. Round to the hundredth place.

For Problem 18 and 19, solve for *x* in the exponential equations.

18. $e^x \approx 13$

19. $e^x \approx 24$

For Problem 20, solve the word problem.

20. To solve the logarithmic equation $\ln x = 5$, write it in exponential form, $e^{\ln x} = e^5$. Since $lnx = log_e x$, then $e^{\ln x} = x$. Use this information to solve $e^{\ln x} = e^5$ for x.

Section 8.14 Module Review

For Problem 1 and 2, evaluate the exponential function for the given x value.

1. $f(x) = 4(0.8)^x$ when $x = 3$

2. $g(x) = 2.1(\frac{1}{3})^x$ when $x = 0$

For Problem 3, solve the multiple-choice problem.

3. Which of the choices represents an increasing function or exponential growth?

a) $f(x) = 3(3)^x$

b) $g(x) = 5(\frac{1}{2})^x$

c) $f(x) = \frac{1}{2}(\frac{1}{3})^x$

d) $g(x) = 0.6(0.95)^x$

For Problem 4 and 5, solve for b in the equations. Use reverse thinking.

4. $48 = 3(b)^4$

5. $0.25 = 2(b)^3$

For Problem 6 and 7, use the formula $A = P(1 + \frac{r}{n})^{nt}$ to solve the compound interest and population problem.

6. Bradon deposited \$450 in a savings account that pays 1.75% annual interest that is compounded quarterly. How much money is in the account after 5 years?

7. The number of young men that play recreational sports in a Young Men's Christian Association is growing by 3% each year. In the year 2015, 220 young men played recreational sports there. Given the percentage of increase does not change, about how many young men will play recreational sports in the program by the year 2020?

For Problem 8 and 9, solve for a in the equation. Use reverse thinking.

8. $4.5 = a(3)^2$

9. $13.5 = a(1.5)^3$

For Problem 10-15, solve the equation for x. Use Properties of Logarithms.

10. $5^x = 25$

11. $\log_2 x = 5.1$

12. $\log_{10} 100 = x$

13. $\log_2 2^3 = x$

14. $\log_x 27 = 3$

15. $3^{(3x+1)} = 81$

For Problem 16 and 17, use Properties of Logarithms to compress the expanded logarithm.

16. $\log_2 4 - \log_2 5$

17. $\log_3 3 + \log_3 4.1$

For Problem 18 and 19, use Properties of Logarithms to expand the compressed logarithms.

18. $\log_{10}(\frac{a}{b})$

19. $\log_3 2x^5$

For Problem 20, write the exponential equation as a natural logarithm and solve for x. Round to the hundredths place.

20. $e^x \approx 20$

Section 8.15 Module Test

For Problem 1 and 2, evaluate the exponential function for the given x value.

1. $h(x) = \frac{1}{2}(3)^x$ when $x = 1$

2. $f(x) = 0.34(5)^x$ when $x = 5$

For Problem 3, solve the multiple-choice problem.

3. Which of the following choices represents a decreasing function or exponential decay?

a) $f(x) = 5(\frac{1}{2})^x$

b) $g(x) = 2(3.4)^x$

c) $f(x) = \frac{1}{3}(4)^x$

d) $g(x) = 0.93(5)^x$

For Problem 4 and 5, solve for b in the equation. Use reverse thinking.

4. $8 = 0.5(b)^2$

5. $2{,}592 = 2(b)^4$

For Problem 6 and 7, use the formula $A = P(1 + \frac{r}{n})^{nt}$ to solve the compound interest and population problem.

6. Mosley deposited \$225 in a savings account that pays 3.5% annual interest that is compounded bi-annually. How much does he have in the account after 5 years?

7. There are 3,052 bacteria in a colony at the beginning of the week (Sunday). How many bacteria will be in the colony by the end of the week (Saturday) if the bacteria are decreasing at a rate of 3.1 percent?

For Problem 8 and 9, solve for a in the equation. Use reverse thinking.

8. $0.1875 = a(0.25)^2$

9. $364.5 = a(0.25)^2$

For Problem 10-15, solve the equation for x. Use Properties of Logarithms.

10. $4^x = 4{,}096$

11. $\log_{3.1} x = 4$

12. $\log_{10} 1 = x$

13. $\log_5 5^2 = x$

14. $\log_x 32 = 5$

15. $1.5(0.5)^x = 60$

For Problem 16 and 17, use Properties of Logarithms to compress the expanded logarithm.

16. $\log_{2.1} 5 + \log_{2.1} 3.6$

17. $\log_4 3.1 - \log_2 4$

For Problem 18 and 19, use Properties of Logarithms to expand the compressed logarithm.

18. $\log_2(\frac{x}{y})$

19. $\log_4 3x^5$

For Problem 20, write the exponential equation as a natural logarithm and solve for x. Round to the hundredths place.

20. $e^x \approx 54$