Building tensorial models of dense suspensions

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Summary

- We need an elastic stress aware of the microstructure to capture shear jamming and thickening
- Relaxation of the microstructure is necessary to obtain rate-dependent effects
- From transient flows we obtain important information to identify different stress contributions
- The dissipative stress needs to take into account contact friction but also details of lubrication

Evolving the microstructure

We denote by $\varphi(\boldsymbol{X}, t)$ the mapping that sends material points $\boldsymbol{X}$ into their position at time t and by $\tilde{\varphi}(\boldsymbol{x}, t)$ its spatial inverse.

The deformation gradient is $\hat{\mathbf{F}}(\boldsymbol{X}, t) := \operatorname{Grad}\varphi(\boldsymbol{X}, t)$ and its spatial counterpart $\mathbf{F}(\boldsymbol{x}, t) := \hat{\mathbf{F}}(\tilde{\varphi}(\boldsymbol{x}, t), t)$ evolves with

$$\frac{\partial \mathbf{F}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\mathbf{F} = (\nabla\boldsymbol{u})\mathbf{F}.$$

We introduce a tensorial field $\mathbf{F}_{\mathrm{R}}(\boldsymbol{x}, t)$ that describes a *relaxed* microstructure, while the *current* microstructure is described by

$$\mathbf{F}_{\mathrm{mic}} := \mathbf{F}\mathbf{F}_{\mathrm{R}}^{-1}, \qquad \mathbf{B}_{\mathrm{mic}} := \mathbf{F}_{\mathrm{mic}}\mathbf{F}_{\mathrm{mic}}^{\mathsf{T}}, \qquad \mathbf{C}_{\mathrm{mic}} := \mathbf{F}_{\mathrm{mic}}^{\mathsf{T}}\mathbf{F}_{\mathrm{mic}}.$$

We *postulate* the evolution equation

$$\frac{\partial \mathbf{F}_{\mathrm{R}}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\mathbf{F}_{\mathrm{R}} = \frac{1}{2\tau_{\mathrm{r}}(\ldots)}(\log\mathbf{C}_{\mathrm{mic}})\mathbf{F}_{\mathrm{R}}$$

to obtain a frame-invariant rate that vanishes when the microstructure is fully relaxed and is driven by a possibly varying relaxation time.

Shear thickening and shear jamming

Shear jamming sets in once the microstructure is sufficiently developed and leads to an elastic response with a diverging τ_{r}. We introduce the excess measure

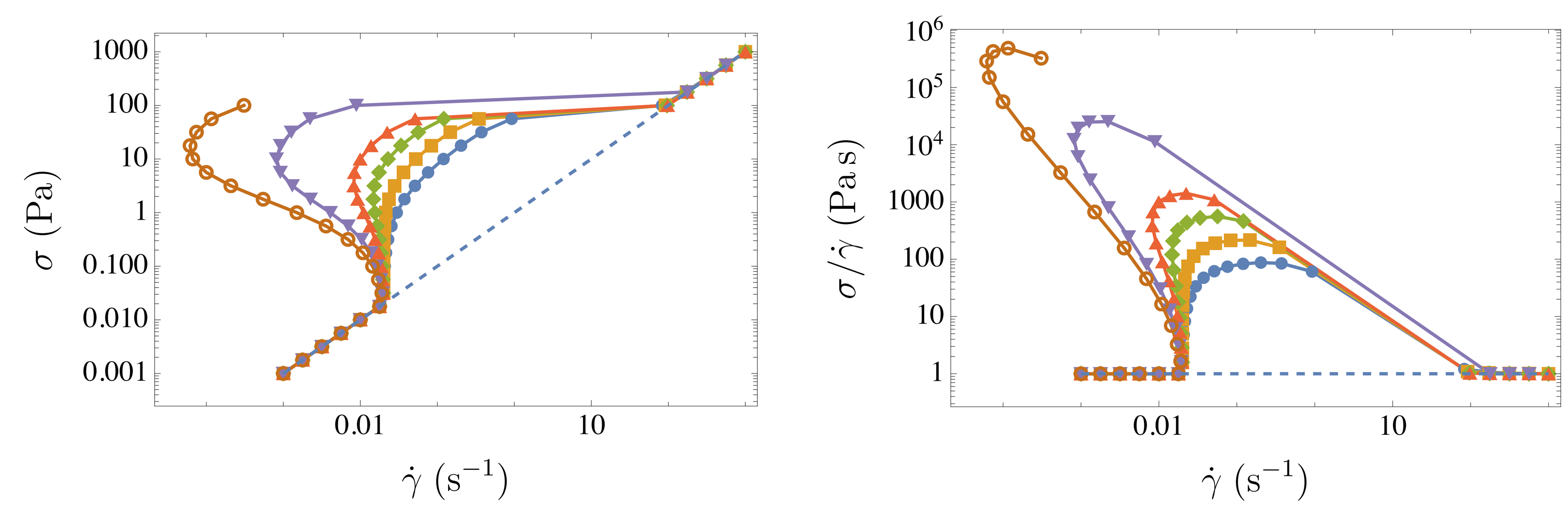
$$\delta = \max\left\{1 - \frac{J}{\|\log\mathbf{B}_{\mathrm{mic}}\|}, 0\right\}.$$

with J a critical build-up of the microstructure and the elastic stress

$$\mathbf{T}_{\mathrm{el}} = (\kappa_1 + \delta\kappa_2)\log\mathbf{B}_{\mathrm{mic}},$$

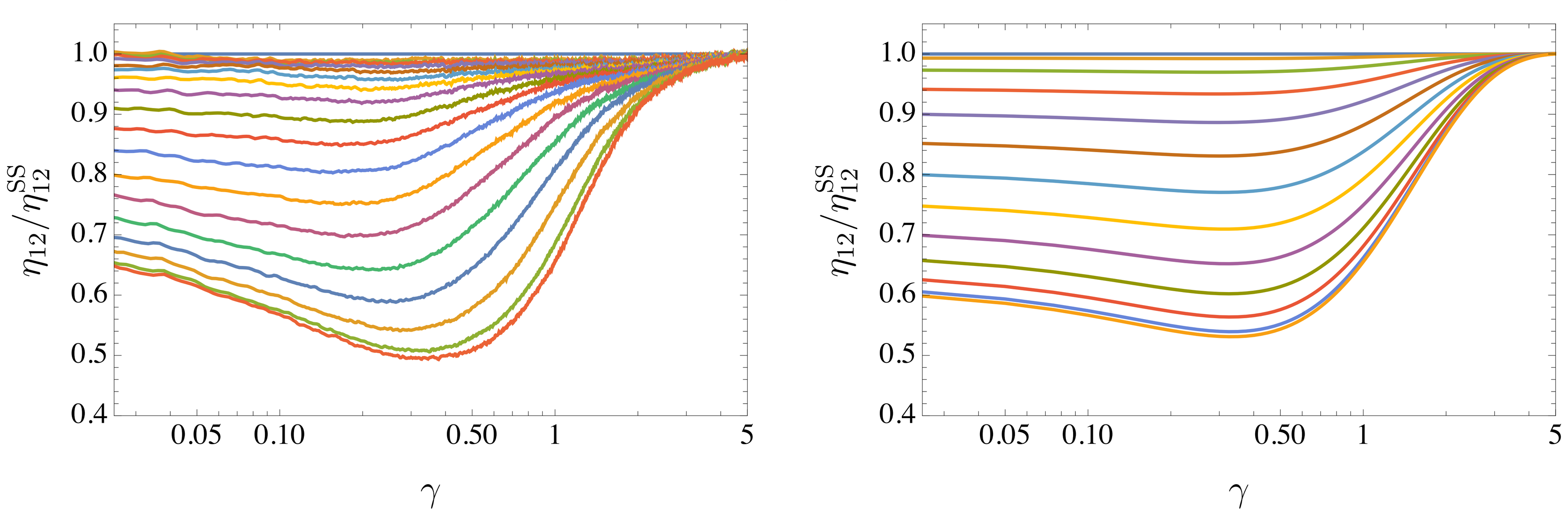
with κ_1 and κ_2 shear moduli.

Setting constant parameters $\tau_{\mathrm{r}}^0 > 0$ and $\alpha > 0$ and assuming $\tau_{\mathrm{r}} = \tau_{\mathrm{r}}^0 \exp(\alpha\delta)$, we obtain shear thickening with an intensity that increases with α.



Anisotropic response in transient flows

An initial microstructure that is not compatible with the steady shear flow is produced by shearing a sample for a given strain and then suddenly rotate the shear plane about the gradient direction by an angle $\theta \in [0, \pi]$. After the rotation, the apparent viscosity η_{12} shows a drop that is larger for larger θ.



The measured behaviour (left) can be reproduced by the model (right) thanks to the elastic term and with an additional dissipation.

Additional dissipation and anisotropy

Transient phenomena are important to suggest the details of a tensorial model. By considering experimental and computational data about shear rotation, published in Blanc *et al.*, PRL 130, 118202 (2023), we see that the elastic contribution is important but not sufficient to capture all effects.

The following dissipative stress includes a proxy for frictional contacts due to the development of micristructure

$$\mathbf{T}_{\mathrm{diss}} = 2\eta\left(1 + \beta_1\|\log\mathbf{B}_{\mathrm{mic}}\|^2\right)\mathbf{D},$$

useful to reproduce the viscosity.

Nevertheless, the transient component $\eta_{32} = \sigma_{32}/\dot{\gamma}$ displays a more complicated behavior. We can capture the contact contribution measured in DEM simulations, but we still miss a term affecting the transient hydrodynamic contribution (see figures below).

$\eta_{32}(\gamma, \theta)/\eta_{12}^{\mathrm{SS}}$ - Experiment | Full DEM data | Model