RIGIDITY DEVELOPMENT IN SHEAR-THICKENING SUSPENSIONS

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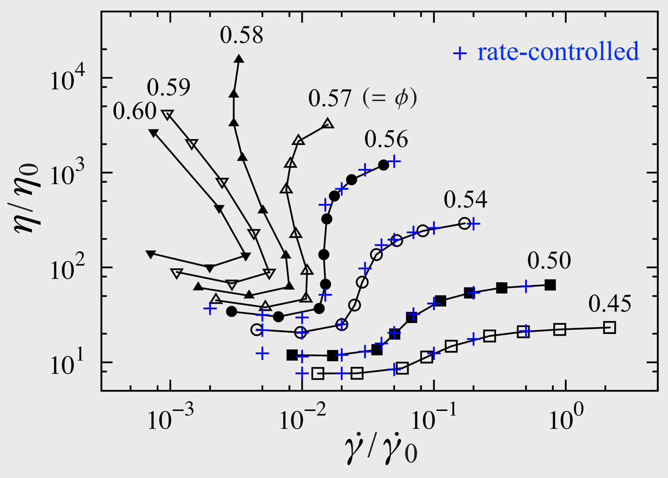
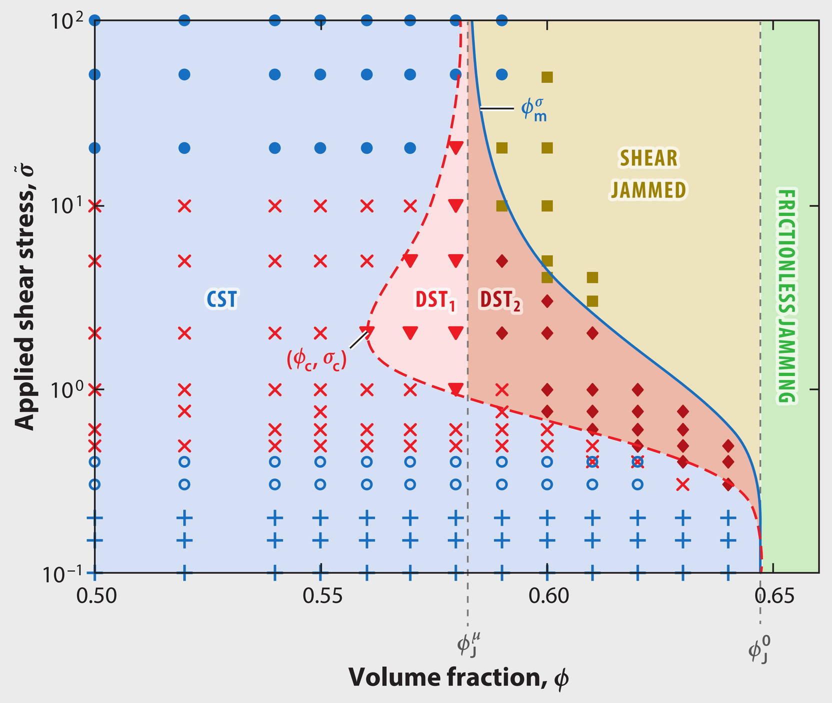
CONTEXT





Top: relative viscosity as a function of the shear rate for different volume fractions. Right: flow-state diagram indicating the different regions in the (volume fraction, shear stress) plane.

Shear-thickening occurs when the shear stress increases more than linearly with the shear rate. As the volume fraction increases, the shear-thickening transitions from continuous (CST) to discontinuous (DST), finally reaching shear-jamming. The lubricated-to-frictional scenario argues that below a certain stress the shearing forces are balanced by the repulsive forces. Above this stress, repulsion is unable to keep the surfaces separated and frictional contacts occur. Varying both volume fraction and shear stress, one can obtain the flow-state diagram.

METHOD

SIMULATIONS: LUBRICATION FLOW DISCRETE ELEMENT METHOD (LF DEM)

Frictional contact force: $|\mathbf{F}^{ij}_{c,t}| \le \mu |\mathbf{F}^{ij}_{c,n}|$

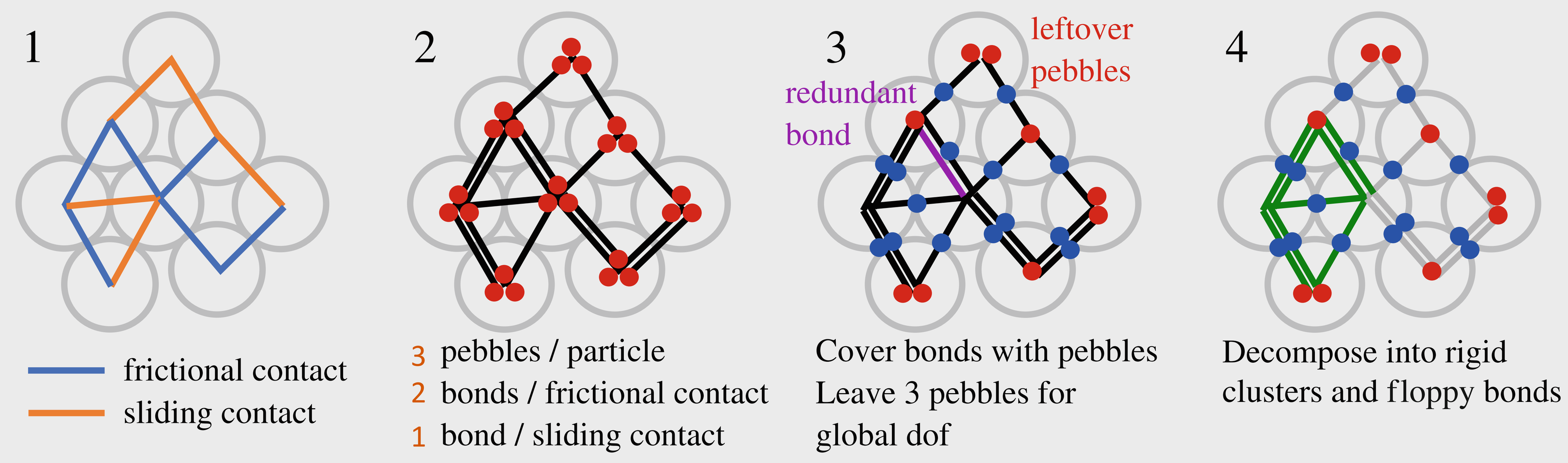
Repulsive potential force: $|\mathbf{F}_r| = \mathbf{F}_0 \exp(-h/\lambda)$

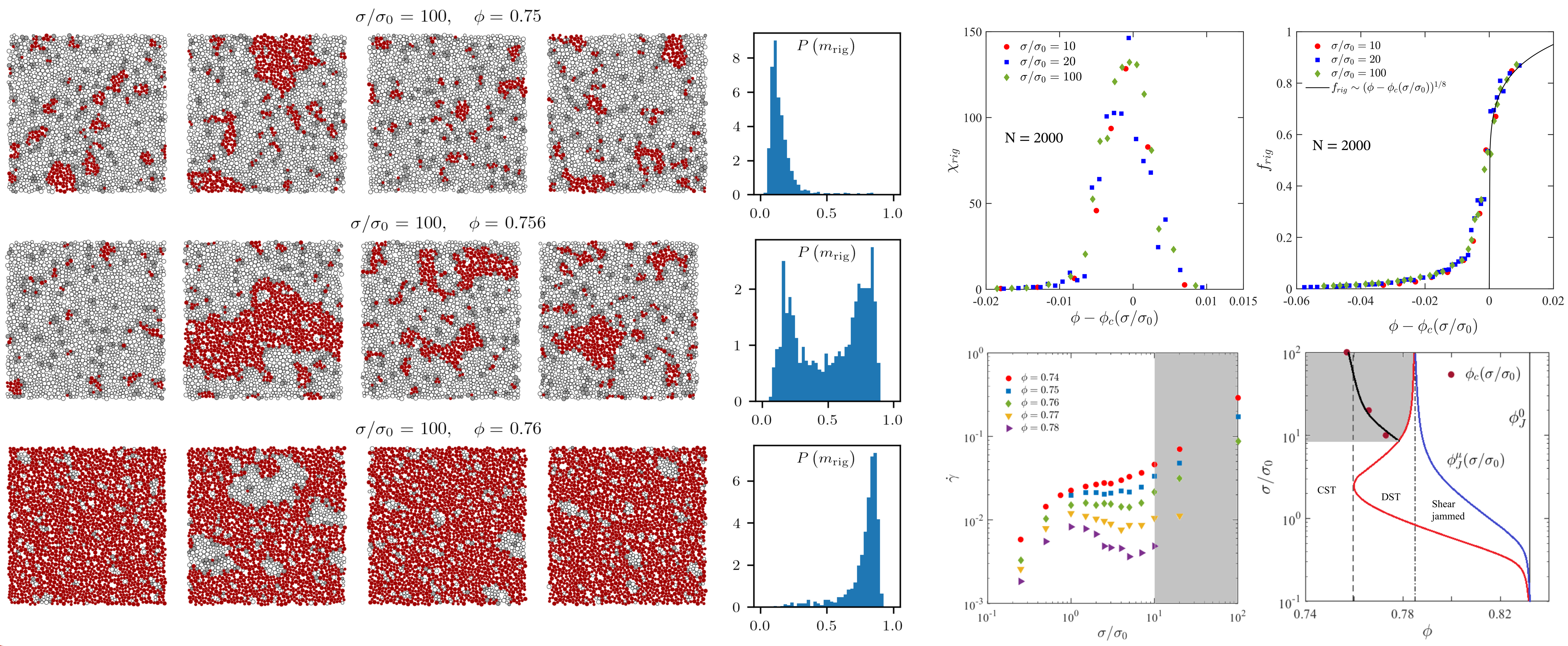
Hydrodynamic force: $\mathbf{F}_h(\mathbf{x},\mathbf{U}_p) = -\mathscr{R}_{FU}(\mathbf{x})\cdot(\mathbf{U}_p - \mathbf{U}_\infty) + \mathscr{R}_{FE}(\mathbf{x}) : \mathbf{E}_\infty$

Force and torque balance: $\mathbf{F}_h(\mathbf{x},\mathbf{U}_p) + \mathbf{F}_c(\mathbf{x}) + \mathbf{F}_r(\mathbf{x}) = \mathbf{0}\,, \quad \mathbf{T}_h(\mathbf{x},\mathbf{U}_p) + \mathbf{T}_c(\mathbf{x}) + \mathbf{T}_r(\mathbf{x}) = \mathbf{0}$

Stress-controlled algorithm: $\dot\gamma = \dfrac{\sigma - \sigma_r - \sigma_c}{\eta_0(1+2.5\phi) + \eta_h}\,, \quad \mathbf{U}_p = \mathbf{U}_\infty + \mathscr{R}_{FU}^{-1}\cdot(\mathscr{R}_{FE} : \mathbf{E}_\infty + \mathbf{F}_r + \mathbf{F}_c)$

IDENTIFYING THE RIGID CLUSTERS: (3,3) PEBBLE GAME





F_{rig} : average number of particles in rigid clusters

$\chi_{rig} = \mathrm{var}(F_{rig})$

ϕ : volume fraction

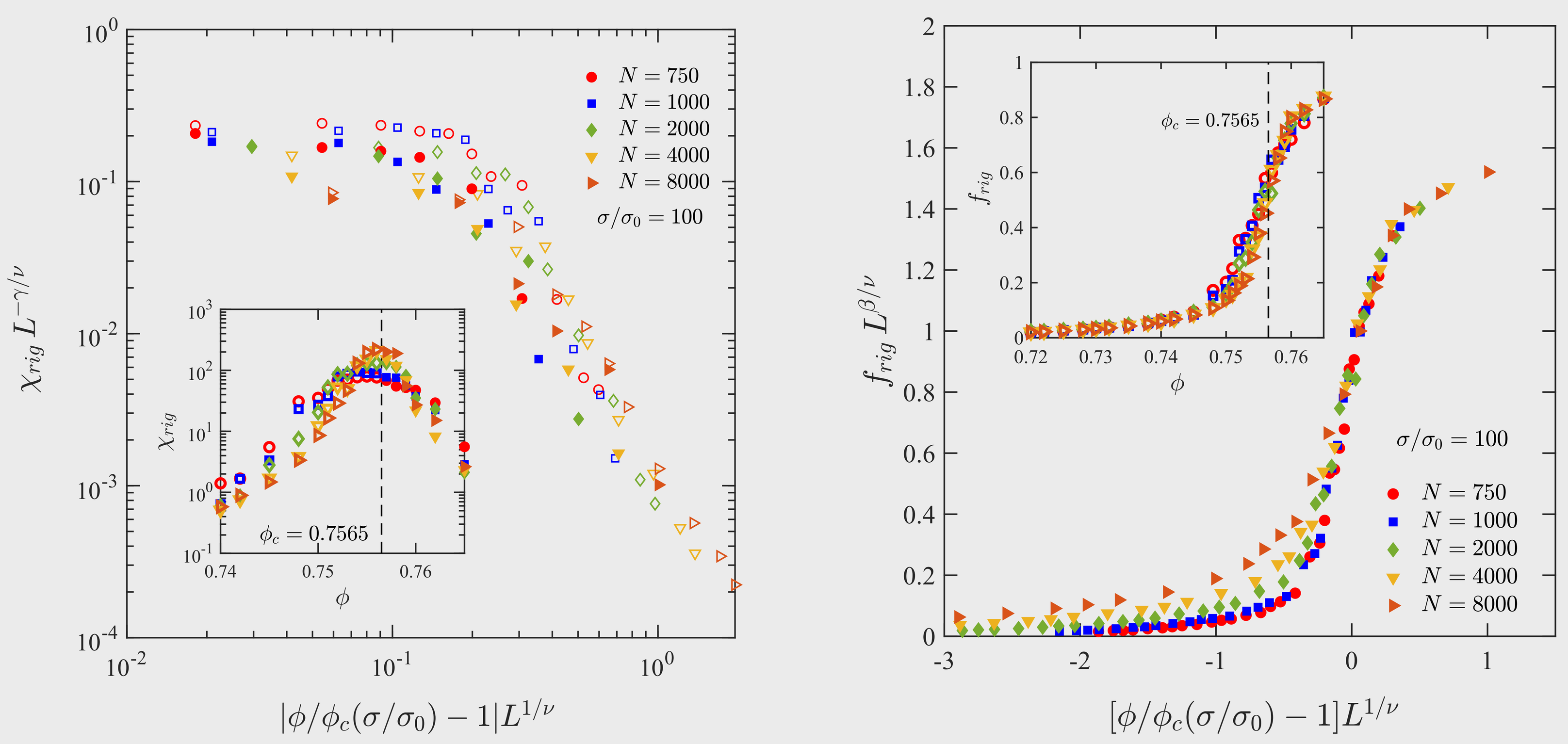
ϕ_c : critical fraction

$f_{rig} = F_{rig}/N \quad (0 \le f_{rig} \le 1)$

The critical packing fraction increases as the stress decreases until the onset of DST is reached. Hence, at high stresses, DST and shear-jamming occur in a phase where particles in contact form rigid clusters.

2D ISING-MODEL UNIVERSALITY CLASS

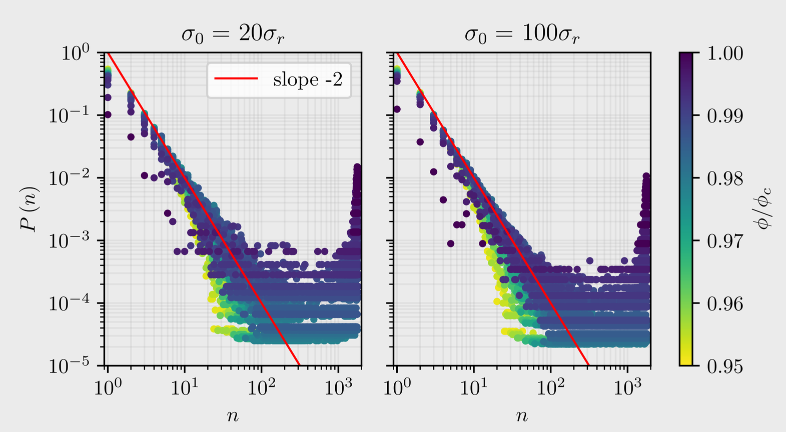
2D Ising exponents: $\beta = 1/8, \quad \nu = 1, \quad \gamma = 7/4$



These are the results obtained for the highest stress varying the system size. In particular, the number of the particles varies from 750 to 8000 particles. On the left, the variance of the average number of particles in rigid clusters is shown, and on the right is the ratio of particles in rigid clusters to the total number of particles. L is the side length of the simulation cell.

By fitting the results, we show that they fall in the 2D Ising-model universality class.

CLUSTER SIZE DISTRIBUTION



The cluster size distribution for two stresses for N = 2000 is shown. As we can notice, bigger clusters form as the critical volume fraction is approached.

CONCLUSIONS

- We have examined the statistical properties of the rigid clusters as a function of solid fraction in a suspension that undergoes the lubricated-to-frictional rheological transition, considering stresses well above the DST transition.
- Approaching the critical packing fraction from below, the suspension becomes unstable to the growth of rigid clusters. The transition quantitatively agrees in its near-critical behavior with the 2D Ising spin system.
- We have shown that the behavior extends over a range of applied stresses. This implies that there is a line of critical points determined by the distance from jamming.

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