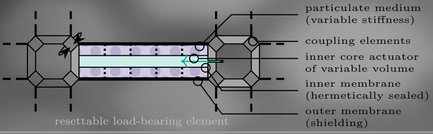
Jammed-granulate metamaterials for soft robotics

Holger Götz **Olfa D'Angelo*** Thorsten Pöschel

Institute for Multiscale Simulation, Universität Erlangen-Nürnberg, Cauerstraße 3, 91058 Erlangen, Germany

INTRODUCTION

soft robotics	robots composed of compliant materials
jamming-based actuators	large stiffness range, minimal volume variation
metamaterials	materials with engineered properties, drawn from that of *constituents* & *structural features*



resettable load-bearing element

application: constitutive element of deployable, self-healing structure

image: ESA/Foster+Partners

METHODS

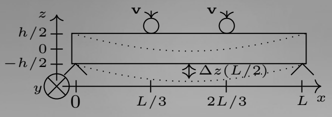
SIMULATION SETUP

DEM simulation, details [PRR **6**, 013061 (2024)]: contact model [PRE **53**, 5382 (1996)], no cohesion, $d = 3.5\,\mathrm{mm} \pm 20\%$, $\sim 11\,000$ particles

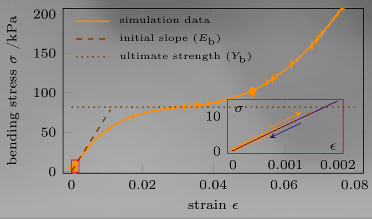
beam geometry: $L = 40\,\mathrm{cm}$, cross section $4\,\mathrm{cm} \times 8\,\mathrm{cm}$

quasi-static deformation ($v = 0.2\,\mathrm{mm\,s^{-1}}$), controlled by deformation

Euler-Bernoulli beam theory: $\sigma = F\frac{L}{h^2 d}$, $\epsilon = \Delta z \frac{108h}{23L^2}$



METHODS



PROTOCOLS

PROTOCOL A: packing density ϕ_{bend} function of friction μ_{p}

$\mu \equiv \mu_{\mathrm{init}} = \mu_{\mathrm{p}}$

seeding → growth until $\phi \geq \phi_{\mathrm{init}}$ → confining pressure → bending starts $\phi_{\mathrm{bend}}(\mu_{\mathrm{p}})$

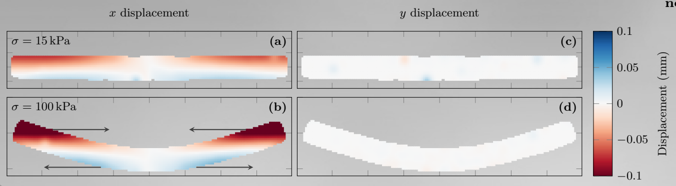
PROTOCOL B: fixed density ϕ_{bend} at bending onset $\forall \mu_{\mathrm{p}}$

$\mu \equiv \mu_{\mathrm{init}}$ $\mu = \mu_{\mathrm{p}}$

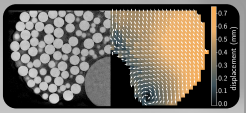
seeding → growth until $\phi \geq \phi_{\mathrm{init}}$ → confining pressure → bending starts $\phi_{\mathrm{bend}}(\mu_{\mathrm{init}})$

step I step II step III step IV

NEUTRAL PLANE LOCATION



neutral plane: no stress, no deformation surface (midplane)



TEMPORAL EVOLUTION

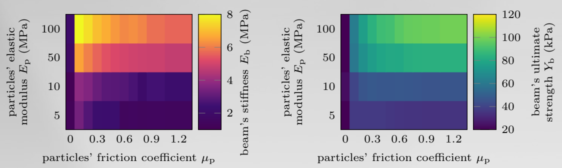
- *early* (before Y_{b}): initially, neutral plane & geometrical midplane coincide
- *throughout bending*: plane **shifts upwards**
- *later stage* (past Y_{b}): extremities bend inwards, shift exacerbated

SPATIAL LOCATION

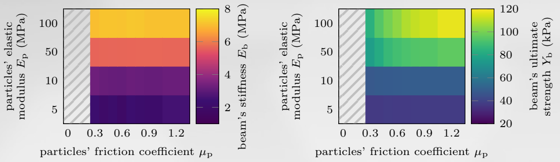
- *above:* compression, particles' motion towards beam's center
- *below:* tension, particles' motion away from beam's center
- *extremities:* largest displacement at extremities (*cf.* granular gripper X-ray CT: more geometric freedom and direct contact with membrane → preferential retraction in surface)

MICRO TO MACRO

PROTOCOL A: ϕ_{bend} function of μ_{p}



PROTOCOL B: $\forall \mu_{\mathrm{p}}$, same ϕ_{bend}

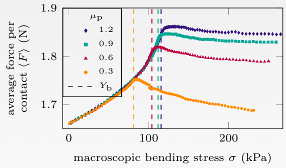


BEAM'S ELASTIC MODULUS

- **depends on preparation through packing density ϕ**
- at fixed ϕ, independent of particles' friction μ_{p}
- generally increases with particles' stiffness, E_{p}
- proportionality relation: beams' elastic modulus $E_{\mathrm{b}} \sim E_{\mathrm{m}}\,\Delta p\,E_{\mathrm{p}}$, membrane stiffness, E_{m}, confining pressure, Δp, particles' stiffness E_{p} [Granul. Matter **25**, 58 (2023)]

BEAM'S ULTIMATE STRENGTH

- increases with E_{p}, μ_{p} and ϕ: at fixed E_{p}, $Y_{\mathrm{b}}(\mu_{\mathrm{p}}) = (Y_{\mathrm{b}}^0 - Y_{\mathrm{b}}^\infty)e^{-\alpha\mu_{\mathrm{p}}} + Y_{\mathrm{b}}^\infty$
- **corresponds to force network collapse:**



STRUCTURAL FEATURES

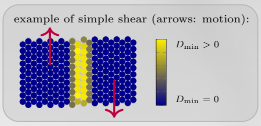
DEFORMATION MECHANISMS

- *affine:* large-scale, homogeneous, coordinated motion; $D_{\min} \to 0$
- *localized:* one particle breaks neighbors-formed cage; $D_{\min} > 0$

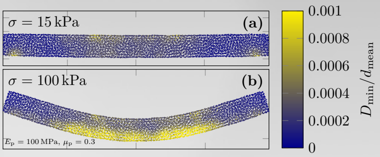
quantified by $D_{\min}$ [PRE **57**, 7192 (1998)]: mean-square deviation from affine of particle i to neighbors j, within $\Delta t = t' - t = 2.5\,\mathrm{s}$:

$$D^2(i,\,t,\,t') = \tfrac{1}{N}\sum_j \Big[\mathbf{r}_j(t') - \mathbf{r}_i(t') - (\mathbb{1} + \epsilon)(\mathbf{r}_j(t) - \mathbf{r}_i(t))\Big]^2,$$

for affine deformation ϵ so that $D_{\min} = \min_\epsilon |D|$

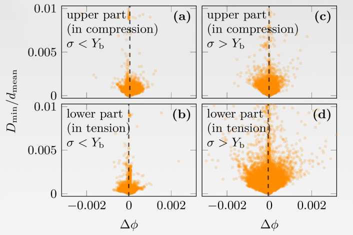


example of simple shear (arrows: motion):

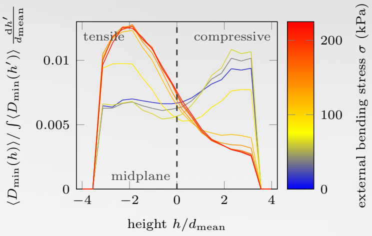


- jammed phase **deforms in affine motion**: no shear band, initially no delimitation at neutral plane

LOCAL DENSITY VARIATIONS



SPATIAL DISTRIBUTION OF LOCAL DEFORMATION

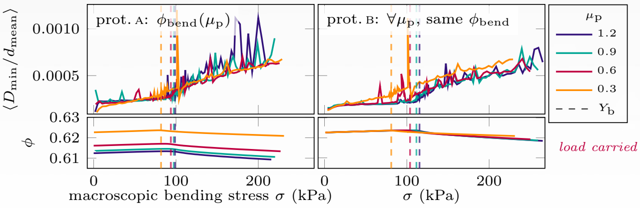


- **deviation from affine is correlated with change in local packing fraction** (V-shape of all graphs)
- past Y_{b}: part in tension (bottom) allows expansion, hence more individual particle motion

plot of normalized $D_{\min}$ as a function of beam's height:

- initially: $D_{\min}$ larger in top part (compressive)
- then becomes much larger in bottom part (tensile), as particles have enough space to move
- transition happens around Y_{b}

BENDING REGIMES



before Y_{b}:

- system densifies on average
- deforms in fully affine manner
- top part compressed, while pressure gradually relieved in bottom (without motion): $D_{\min}$ close to 0

after Y_{b}:

- as neutral plane rises, part in tension increases
- ϕ on average decreases, effectively leaving more space for particles' motion: $D_{\min}$ increases

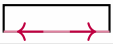
effect of particles' properties:

- μ_{p}: influence through initial packing density but <u>not</u> deformation mechanism
- μ_{p}: shifts Y_{b} for hard particles; no influence on Y_{b} for soft particles
- soft particles (not shown): higher packing density at same confining pressure, hampers non-affine deformation

load carried by:

granular packing (top)
in compression

external membrane (bottom)
in tension



CONCLUSION

preparation protocol:

- select E_{p} for desired compliance
- maximize packing density, e.g. preconstrain beam in pre-Y_{b} regime

reinitialization of beam:

- *in-situ* sensing for optimizing reinitialization frequency: find Y_{b}, when granulates stop carrying load (compression), possibly through extension of bottom membrane

H. Götz, T. Pöschel, O. D'Angelo, Phys. Rev. Research **6**, 013061(2024)
O. D'Angelo, A. Sack, T. Pöschel, German Patent App. DE 10 2023 130 608.1 (2023)
A. Santarossa, O. D'Angelo, A. Sack, T. Pöschel, Granul. Matter **25**, 16 (2023)

*olfa.dangelo@fau.de