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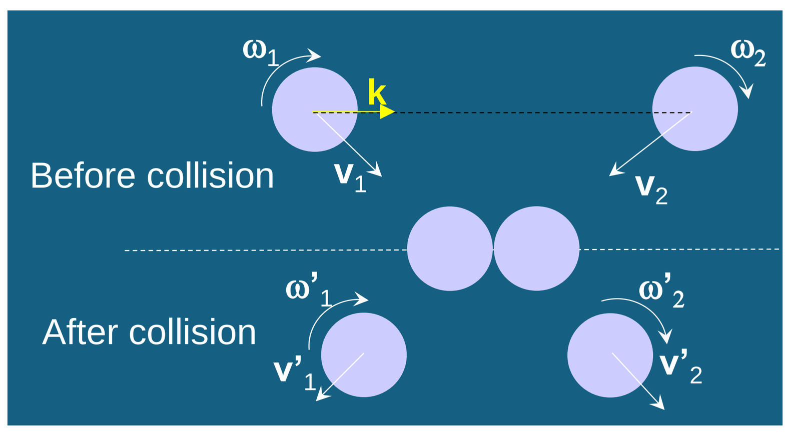
On granular flows: from kinetic theory to inertial rheology and nonlocal constitutive models

Diego Berzi, Phys. Rev. Fluids, 9, 034304 (2024)

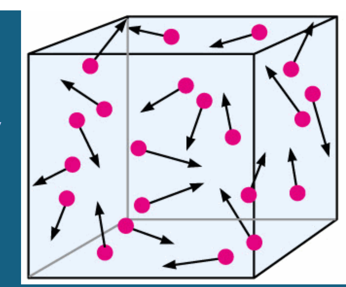
POLITECNICO MILANO 1863



Collisions can be sticking or sliding

$e_n \le 1$ normal coefficient of restitution
$e_t \le 1$ tangential coefficient of restitution
$\mu \ge 0$ surface friction

Origin of stresses for granular gases: transfer of momentum associated with velocity fluctuations

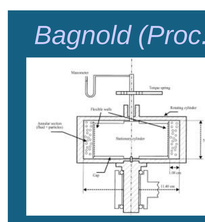


Granular temperature T
one third of the mean square of velocity fluctuations *(Ogawa 1978)*

$$\text{pressure} \propto \frac{\text{momentum} \times \text{frequency}}{\text{area}} = \frac{mT^{1/2} \times T^{1/2}/d}{d^2} \propto \rho_p T$$

$$\text{shear stress} \propto \frac{\text{momentum} \times \text{frequency}}{\text{area}} = \frac{md\dot{\gamma} \times T^{1/2}/d}{d^2} \propto \rho_p d T^{1/2} \dot{\gamma}$$

Bagnold (Proc. R. Soc. Lond. A 1954)



$$\text{stresses} \propto \frac{\text{momentum} \times \text{frequency}}{\text{area}} = \frac{md\dot{\gamma} \times \dot{\gamma}}{d^2} \to \begin{array}{l} p = f(\nu)\rho_p d^2 \dot{\gamma}^2 \\ s = \tan\alpha_B p = \tan\alpha_B f(\nu)\rho_p d^2\dot{\gamma}^2 \end{array}$$

Inertial (μ-I) rheology (GdR MiDi EPJE 2004; da Cruz et al. 2005; Jop et al. JFM 2005)

$$\nu = f^{**}\left(I = \frac{\dot{\gamma} d}{\sqrt{p/\rho_p}}\right) \text{(compressible)} \qquad \mu^* = \frac{s}{p} = f^*(I) \quad \to \quad \begin{array}{l} p = f'(\nu)\rho_p d^2\dot{\gamma}^2 \\ s = f''(\nu)\rho_p d^2 \dot{\gamma}^2 \end{array}$$

Kinetic theory of granular gases (e.g., Jenkins and Savage JFM 1983; Garzo and Dufty PRE 1999)

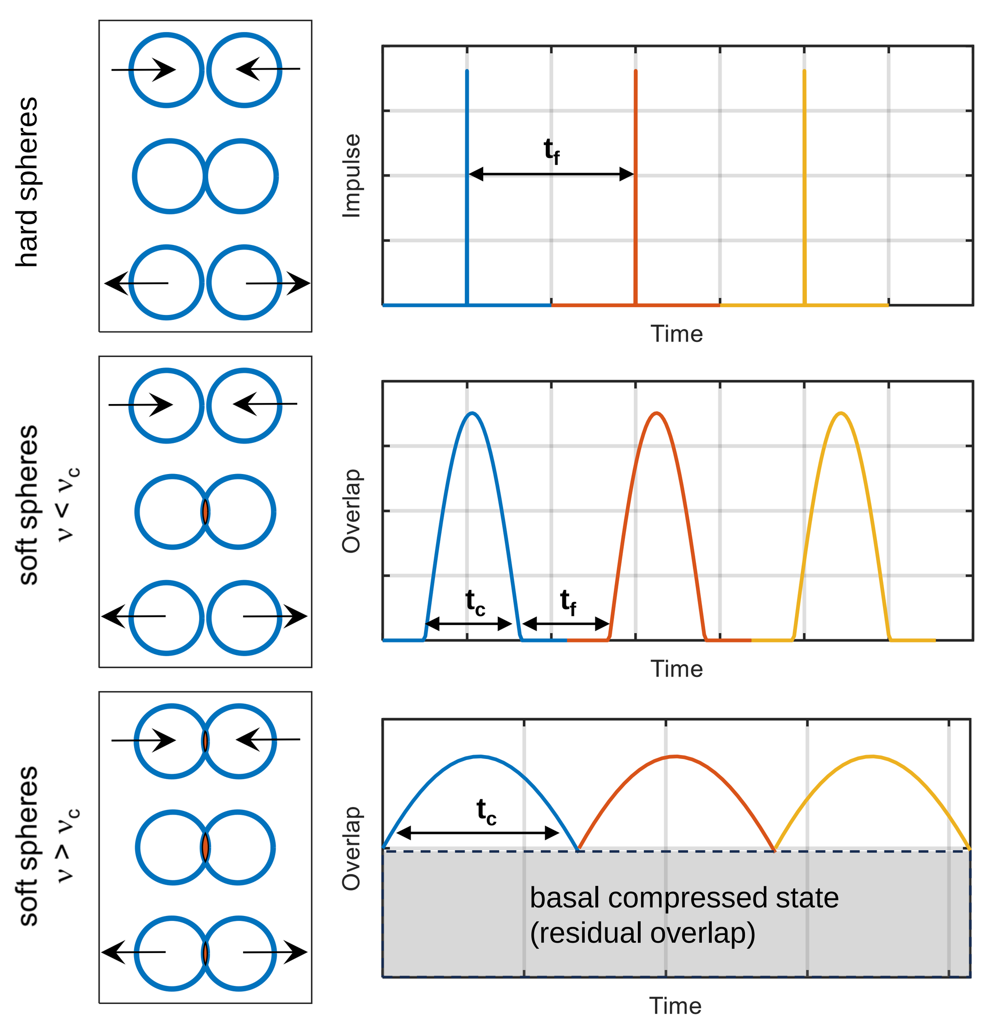
$$p = f_1(\nu)\rho_p T \qquad s = f_2(\nu)\rho_p d T^{1/2}\dot{\gamma}$$

Balance of fluctuation energy
$$\frac{3}{2}\rho_p \nu \frac{DT}{Dt} = -\nabla \cdot \mathbf{Q} + s\dot{\gamma} - f_3(\nu)\frac{\rho_p}{d}T^{3/2}$$
Diffusion, Production, Dissipation

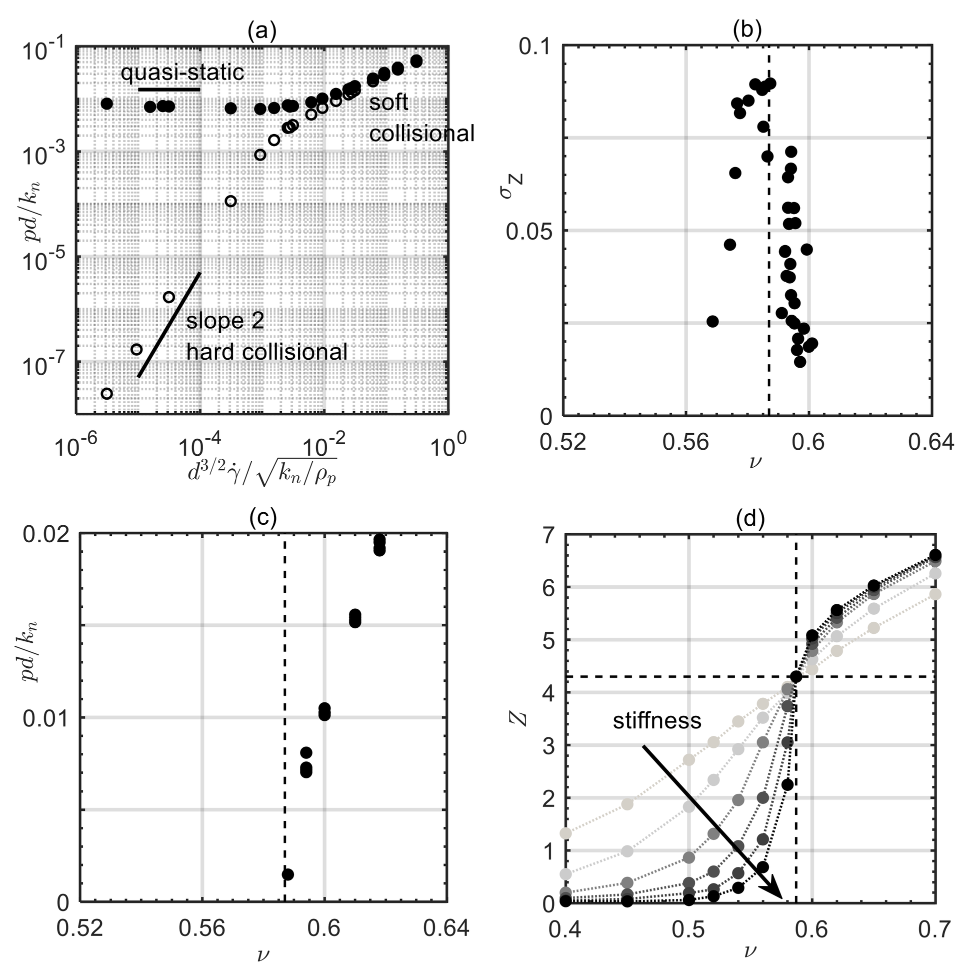
Steady, no diffusion →

$$p = \frac{f_1(\nu) f_2(\nu)}{f_3(\nu)}\rho_p d^2\dot{\gamma}^2 \qquad s = f_2(\nu)\sqrt{\frac{f_2(\nu)}{f_3(\nu)}}\rho_p d^2\dot{\gamma}^2 \qquad s\dot{\gamma} = \Gamma \to T = \frac{f_2(\nu)}{f_3(\nu)} d^2\dot{\gamma}^2$$

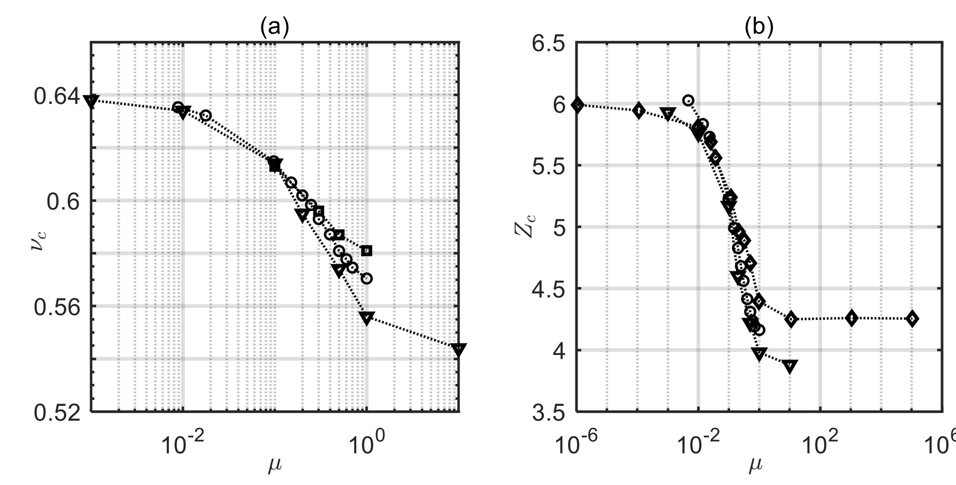
MOMENTUM EXCHANGE AND ORIGIN OF RATE-INDEPENDENT BEHAVIOUR



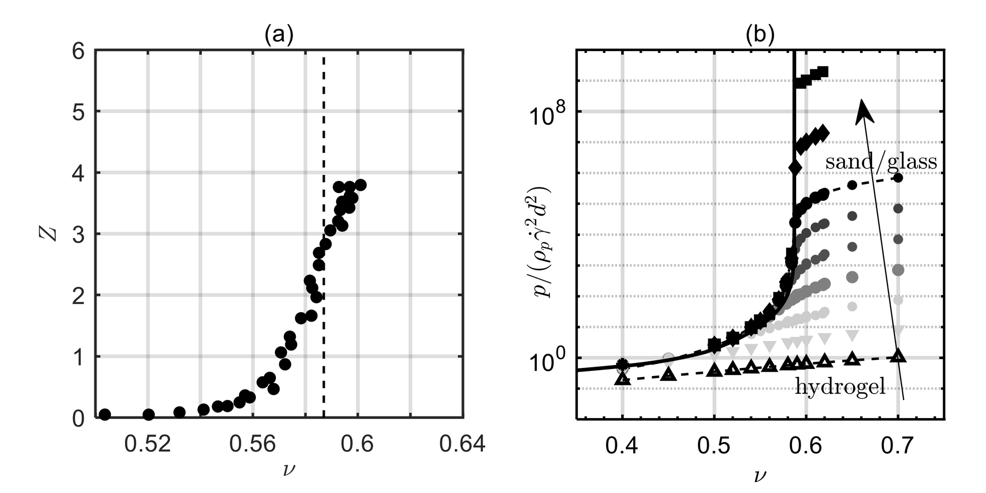
MEASURING THE CRITICAL POINT



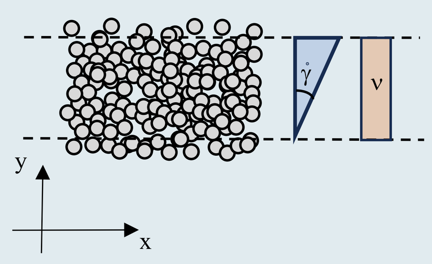
CRITICAL POINT: ROLE OF FRICTION



REALISTIC PARTICLES: BINARY COLLISIONS



STEADY, HOMOGENEOUS FLOWS



Discrete simulations of Chialvo et al PRE 2012, Chialvo and Sundaresan PHF 2013, Vescovi and Luding SM 2016

Inertial (μ-I) rheology

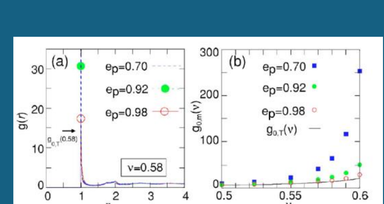
$$\nu = \nu_c - aI \text{ (phenomenological)}$$
$$\mu^* = \mu_s + \frac{\mu_2 - \mu_s}{I_0/I + 1} \text{ (phenomenological)}$$

Input parameters
$$\nu_c, a, \mu_s, \mu_2, I_0$$

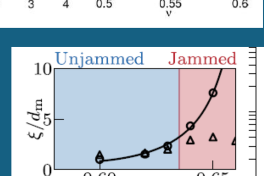
Extended Kinetic Theory (Jenkins and Zhang PHF 2002; Jenkins GM 2007; Berzi AM 2014)

$$p = f_p(e_n, e_t, \mu)\nu\chi_0 \frac{L}{d}\rho_p d^2\dot{\gamma}^2$$
$$s = f_s(e_n, e_t, \mu)\nu\chi_0\sqrt{\frac{L}{d}}\rho_p d^2\dot{\gamma}^2$$
$$T = f_T(e_n, e_t, \mu)\frac{L}{d}d^2\dot{\gamma}^2$$
$$\chi_0 = \chi_0(\nu, \nu_c) \text{ (radial distribution function at contact, phenomenological, but measurable)}$$
$$\frac{L}{d} = f_0(\nu, \nu_{rcp}, e_n, e_t, \mu) \text{ (correlation length, phenomenological, but measurable)}$$

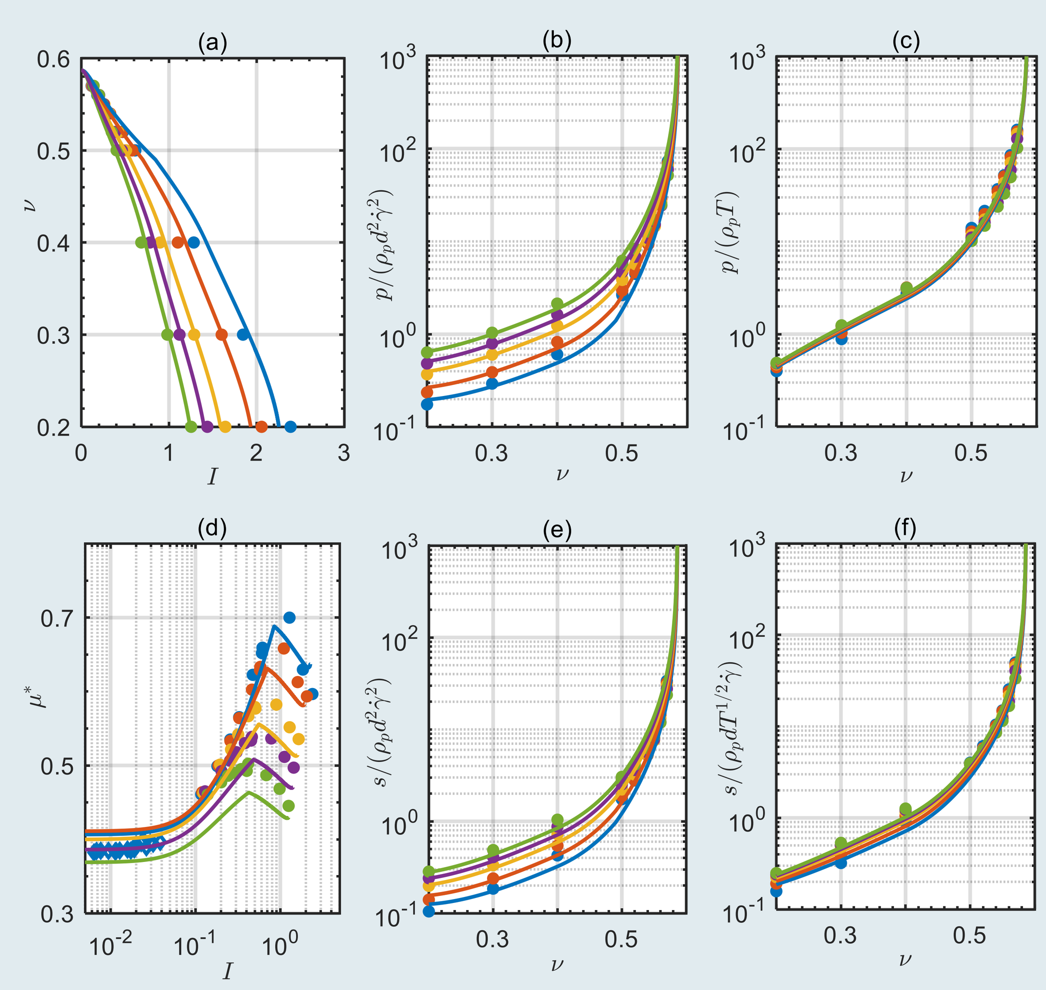
Mitarai and Nakanishi (PRE 2007)



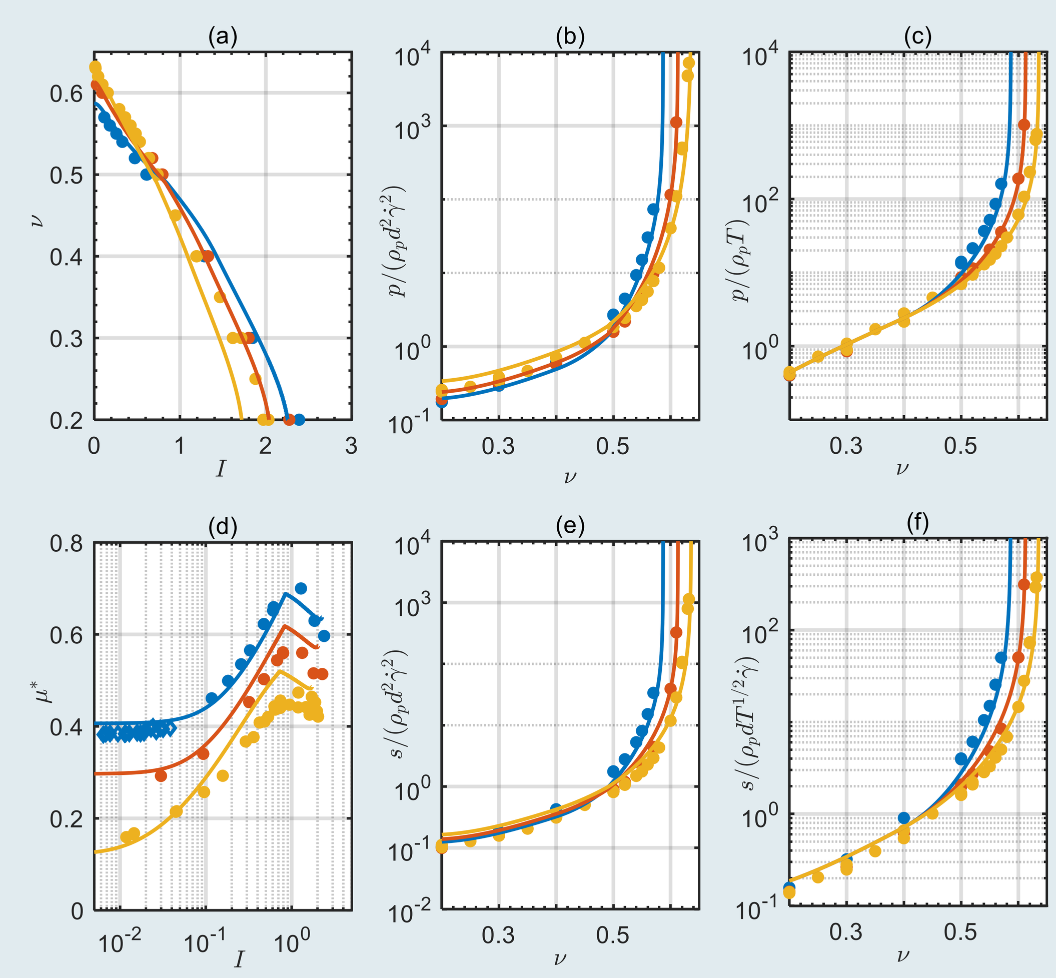
Oyama et al (PRL 2019)



Input parameters
$$\nu_c, \nu_{rcp}, e_n, e_t, \mu$$

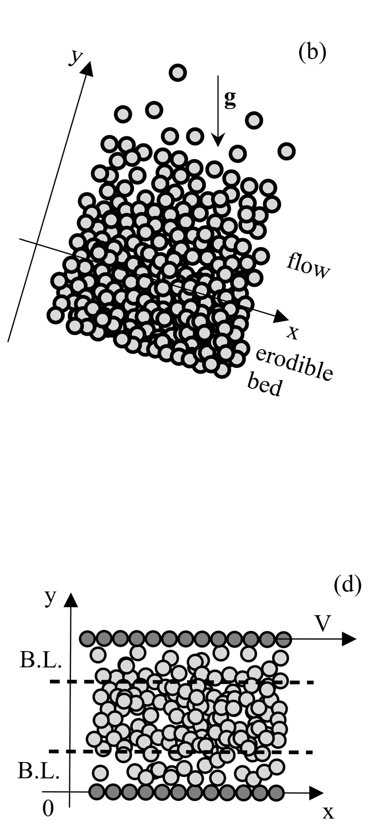


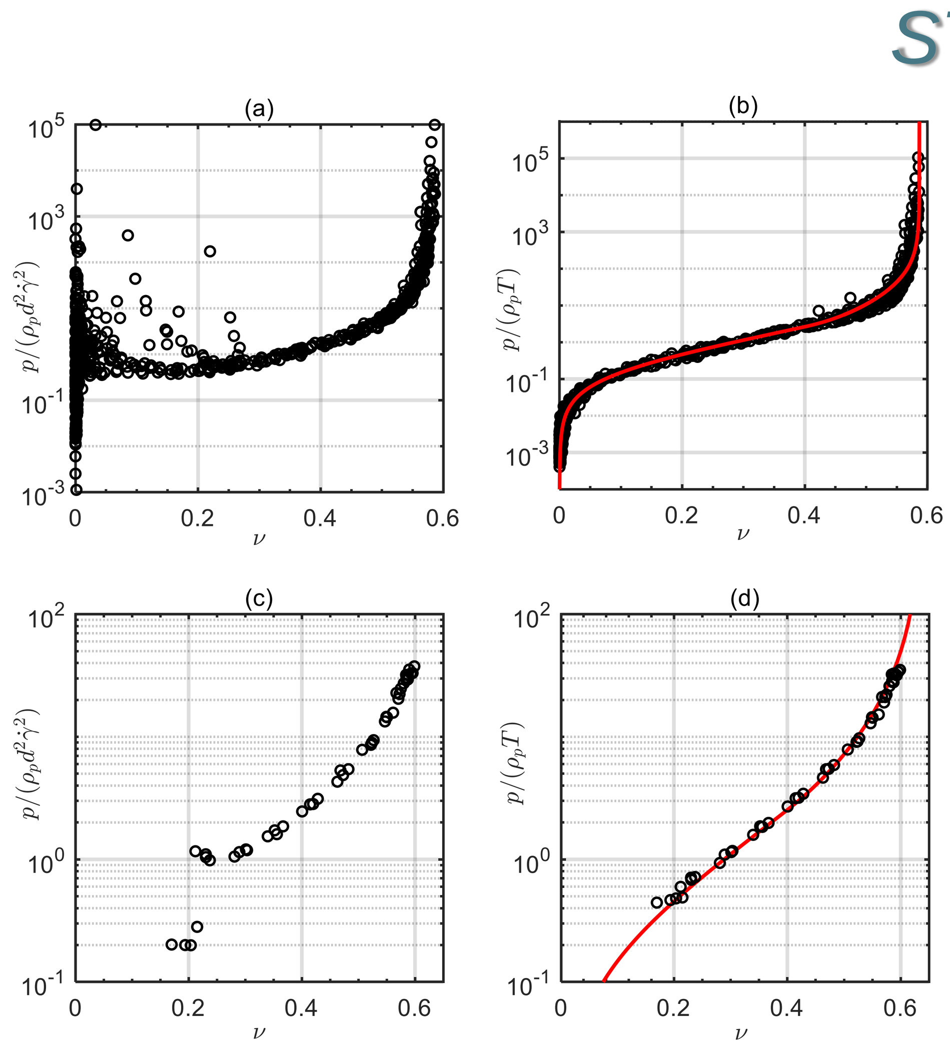
μ=0.5, e_n=0.7, 0.8, 0.9, 0.95, 0.99



e_n=0.7, μ=0, 0.1, 0.5

STEADY, HETEROGENEOUS FLOWS





$$\frac{\dot{\gamma}p}{\Gamma s}\nabla \cdot \left(\rho_p \frac{4M_\infty \nu^2 \chi_0}{\pi^{1/2}} d2T\nabla T^{1/2}\right) = \frac{\dot{\gamma}p}{s} - \frac{\dot{\gamma}p}{\Gamma/\dot{\gamma}},$$
Energy balance of Kinetic Theory

$$g = \frac{\dot{\gamma}p}{s} \approx \frac{5\pi^{1/2}}{2}\frac{T^{1/2}}{d}$$
Granular Fluidity (Kinetic Theory)

$$g_{loc} = \frac{\dot{\gamma}p}{\Gamma/\dot{\gamma}} \approx \frac{\pi^{1/2}(1+e_n)}{6(1-e_{\text{eff}}^2)}\frac{L}{d}\frac{\dot{\gamma}^2 d}{T^{1/2}},$$
Local Granular Fluidity (Kinetic Theory)

$$\frac{1}{3\rho_p(1-e_{\text{eff}}^2)\nu^2\chi_0 T}\frac{M_\infty}{1+e_n}\frac{L}{d}d^2\nabla \cdot (p\nabla g) = g - g_{loc}.$$

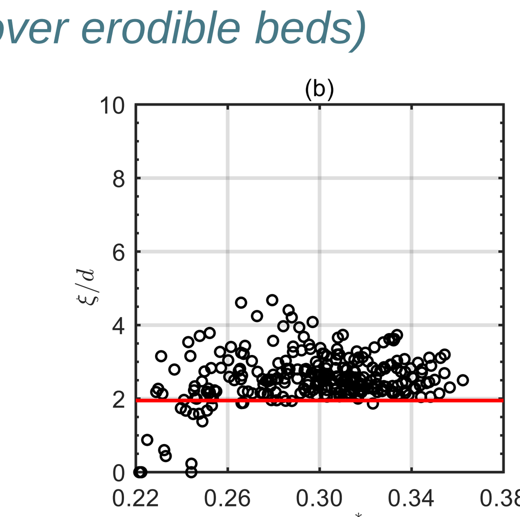
neglecting pressure gradient

$$\frac{2M_\infty}{3(1-e_{\text{eff}}^2)}\frac{L}{d}d^2\nabla^2 g = g - g_{loc},$$

$$\xi^2\nabla^2 g = g - g_{loc},$$
Nonlocal inertial rheology (Kamrin and Koval PRL 2012)

Cooperativity length (Kinetic Theory)
$$\xi = \sqrt{\frac{2M_\infty}{3(1-e_{\text{eff}}^2)}\frac{L}{d}}d.$$

Cooperativity length measured in discrete simulations (inclined flows over erodible beds)



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