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Critical State, Steady Shear and Jamming

Critical-state soil mechanics, steady-state quasi-static shear, and shear jamming are often treated as distinct phenomena, described through different languages and modelling assumptions. In this presentation, we argue that they can be unified within a single statistical-mechanics framework built on Edwards ensembles. The central idea is to view steady-state quasi-static shear as a driven process that continually explores a large set of mechanically stable, jammed microstates. Within this picture, the observable steady state corresponds to a macrostate selected by entropy maximisation over the Edwards ensemble, subject to the relevant macroscopic constraints.

From this variational principle we obtain relations linking volume and pressure, and linking the shear-stress ratio and pressure. These relations reproduce—both in functional form and in quantitative trends—the characteristic critical-state relationships measured in soil mechanics experiments, providing a direct bridge between classical critical-state concepts and jammed-state statistical mechanics.

We further examine conditions under which shear localisation develops. In these cases, bulk-averaged measurements deviate from the homogeneous steady-shear predictions. Strikingly, however, the state variables measured inside the shear band are found to match much more closely the values expected from homogeneous steady shear, suggesting that the shear band acts as the primary region in which the system realises the entropy-maximising steady macrostate.