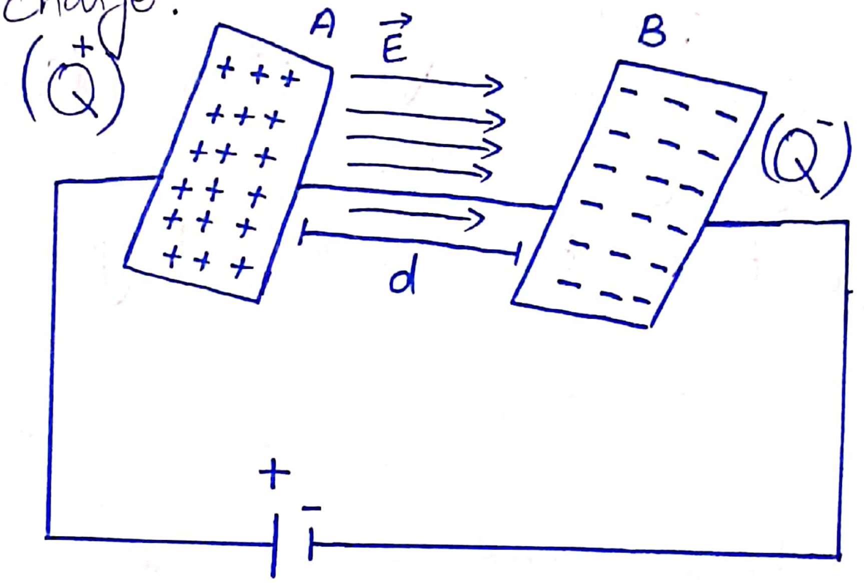
Capacitor :

A device which is used to store the electrical energy and electric charge.



As we know,

$Q \propto V$

$Q = CV$

∴ (C is the constant of proportionality)

$Q = CV$

For C :

$PV = nRT$ (Inert gas equation)
(General gas law)

$n = \frac{PV}{RT}$ —— ①

∴ P = pressure
V = volume
T = temperature

$R = 8.31 \, J \, kg^{-1}$

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T ∵ $Q = CV$ —— (2)

F From eq (1) we get;

$$n = \frac{V}{RT} P \quad \text{—— (3)}$$

Comparing (2) and (3)

Here from analogy

C ⟶ V (volume) (space)

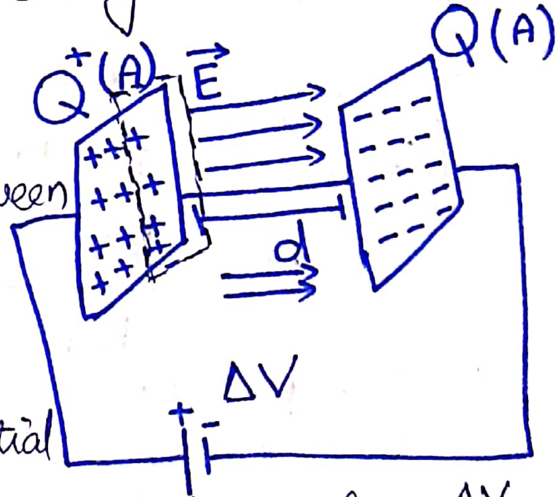
where C is the capacitance of Capacitor to store charge.

Types of Capacitor:

a) Parallel plate capacitor.
b) Cylindrical capacitor.
c) Spherical capacitor.

(a) Parallel plate Capacitor:

Consider a parallel plate capacitor each with area A. The charge on one plate is Q^+ and the charge on other plate is Q^-. The distance between the plates is d and the electric field between them is E. Potential difference between the plates is ΔV.



To calculate electric field between the plates, we take a gausian surface in the form of sheet enclosing positive charges. Then by using gausses law we can write.

$$Q = CV \quad \text{——— (1)}$$

$$C = \frac{Q}{V} \quad \text{——— (2)}$$

For V :

$$V = -\int_{-}^{+} E.ds.$$

$$V = \int_{+}^{-} E.ds. \quad \text{—— (A)}$$

For E :

$$E = E_+ + E_-$$

$$\therefore E = \frac{\sigma}{2\varepsilon_0} \quad \text{Put in above.} \qquad \therefore \sigma = \text{Sigma.}$$

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0} \quad \text{——— (B)}$$

Put (B) in (A) we get,

$$V = \int_{+}^{-} \frac{\sigma}{\varepsilon_0}\, ds$$

$$V = \frac{\sigma}{\varepsilon_0} \int ds$$

$$V = \frac{\sigma}{\varepsilon_0}(d)$$ Put in eq (2)

$$C = \frac{Q\varepsilon_0}{\sigma d}$$

$$\therefore \sigma = \frac{Q}{A}$$

$$C = \frac{Q A \varepsilon_0}{Q d}$$

$$\boxed{C = \frac{A\varepsilon_0}{d}}$$

This is the Capacitance of parallel plate capacitor.

For medium:

$$C = \frac{A\varepsilon}{d}$$

$$\therefore \varepsilon = \varepsilon_r \varepsilon_0$$

$$\boxed{C = \frac{A\varepsilon_r\varepsilon_0}{d}}$$

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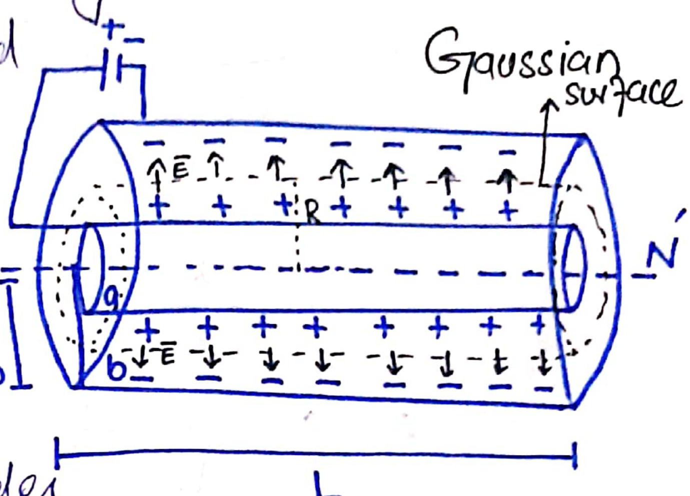
(b) Cylindrical capacitor:

Consider two co-axial cylinders with inner cylinder of radius a and outer cylinder of radius b, and a common axis NN′ connected to a battery. The inner cylinder is connected with positive terminal and outer cylinder is connected with negative terminal of the battery. The electric field lines directed from positive towards negative terminal. To calculate the electric field $\vec{E}$ we take a gaussian surface in the form of cylinder enclosing positive charge we get.



$$Q = CV$$

$$C = \frac{Q}{V} \quad \text{—— (1)}$$

$$V = -\int_b^a E \cdot ds.$$

$$V = \int_a^b E \cdot ds. \quad \text{—— (2)}$$

As here cylinder behaves as line of charge

$$E = \frac{\lambda}{2\pi \varepsilon_0 r} \quad \text{put in above}$$

$$V = \int_a^b \frac{\lambda}{2\pi\varepsilon_0 r}\, dr.$$

$$V = \frac{\lambda}{2\pi\varepsilon_0} \int_a^b \frac{1}{r}\, dr.$$

$$V = \frac{\lambda}{2\pi\varepsilon_0} \left[\ln r\right]_a^b$$

$$V = \frac{\lambda}{2\pi\varepsilon_0} \left(\ln \frac{b}{a}\right)$$

Put in ① we get.

ce

$$C = \frac{Q}{\frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{b}{a}\right)}$$

$\therefore \lambda = \frac{Q}{L} \Rightarrow Q = \lambda L$ (put in above)

$$C = \frac{\lambda L\, 2\pi\varepsilon_0}{\lambda \ln(b/a)}$$

$$\boxed{C = \frac{2\pi\varepsilon_0 L}{\ln(b/a)}}$$

This is the capacitance of cylindrical capacitor.

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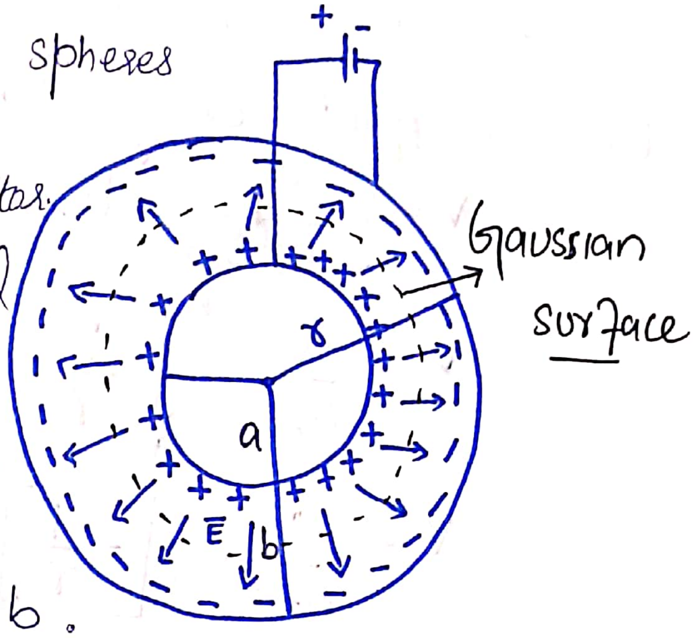
c) Spherical Capacitor:

Two con-centric spheres connected to a battery form a spherical capacitor.



Consider two spherical capacitors or concentric sphere's with inner sphere of radius a and outer sphere of radius b.

The inner sphere is connected with positive terminal and outer sphere is connected to negative terminal of the battery. The electric field lines directed from positive towards negative. To find the electric field $\vec{E}$ we take a gaussian surface in the form of sphere enclosing inner sphere. As we know,

$$Q = CV$$

$$C = \frac{Q}{V} \quad \text{——— (1)}$$

For V:

$$V = -\int_{b}^{a} E.ds.$$

$$V = \int_{a}^{b} E\,ds. \quad \text{——— (2)}$$

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For E :

For shell of charge it behaves like a point charge. $\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}$ Put in eq ② we get.

$$V = \int_a^b \frac{q}{4\pi\varepsilon_0 r^2} \, ds. \quad (ds = dr)$$

$$V = \frac{q}{4\pi\varepsilon_0} \int_a^b \frac{1}{r^2} \, dr.$$

$$V = \frac{q}{4\pi\varepsilon_0} \left[-\frac{1}{r} \right]_a^b$$

$$V = -\frac{q}{4\pi\varepsilon_0} \left[\frac{1}{b} - \frac{1}{a} \right]$$

$$V = -\frac{q}{4\pi\varepsilon_0} \left[\frac{a-b}{ab} \right]$$

$$V = \frac{q}{4\pi\varepsilon_0} \left[\frac{b-a}{ab} \right]$$

Put in eq ① we get,

$$C = \frac{q}{\frac{q}{4\pi\varepsilon_0} \left[\frac{b-a}{ab} \right]}$$

$$\boxed{C = \frac{4\pi\varepsilon_0 [ab]}{[b-a]}}$$

This is the capacitance of Spherical Capacitors.

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