Biot - Savart's Law :

It is for magnetic field.

Statement :

A law which is used, to calculate magnitude of magnetic field at a point around conductor carrying current is called Biot - Savart's Law.

Biot - Savart's law helps calculate the magnetic field around a current carrying wire, coil or any shape, and is a crucial concept in understanding electromagnetism, motors, generators and more!.

Derivation :

As magnetic field varies directly with the speed of charges constituting current and magnitude of charges. On the other hand, it varies inversely with square of the distance.

$$d\bar{B} \propto \frac{dq\,V}{r^2}$$

$$d\bar{B} = \frac{\mu_0}{4\pi} \frac{dq\,V}{r^2} \hat{r} \text{ (unit vector)}$$

$$d\bar{B} = \frac{\mu_0}{4\pi} \frac{dq}{r^2} \bar{V} \times \hat{r}$$

$$V = \frac{dl}{dt}$$

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$$d\bar{B} = \frac{\mu_0}{4\pi} \frac{dq}{r^2} \frac{dl}{dt} \times \hat{r}$$

$$d\bar{B} = \frac{\mu_0}{4\pi r^2} \frac{dq}{dt} dl \times \hat{r} \qquad \because \frac{dq}{dt} = i$$

$$\boxed{d\bar{B} = \frac{\mu_0 i}{4\pi r^2} dl \times \hat{r}}$$

This expression is Biot-Savart's Law :

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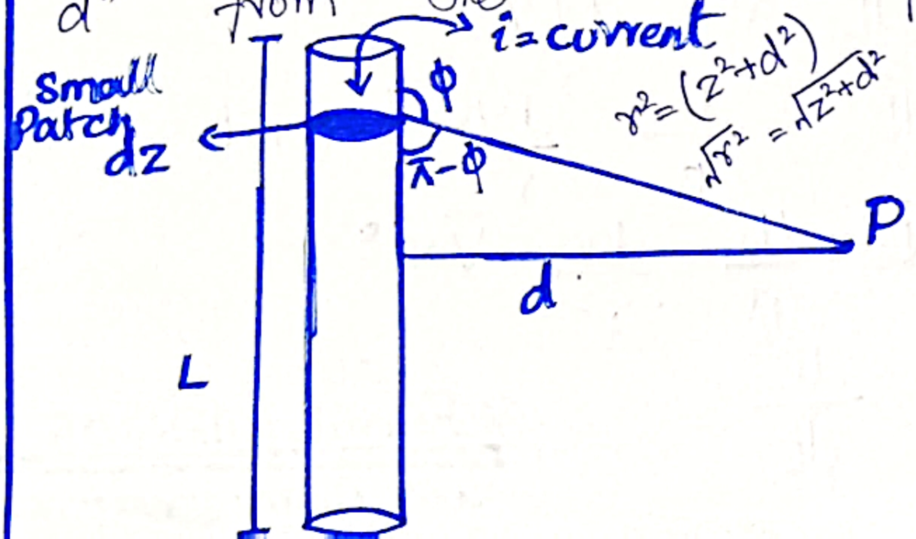
Magnetic Field Due to a Straight Wire Segment (EMT) :

Let we consider a line of wire carrying current "i" and length of the wire "L". The wire lies along the z-axis and we want to determine the magnetic field at a point "P" along y-axis. Distant "d" from the centre of the wire.



∵ From Biot - Savart's Law.

$$d\bar{B} = \frac{\mu_0 i}{4\pi} \frac{dl \times \hat{r}}{r^2}$$

$$d\bar{B} = \frac{\mu_0 i}{4\pi} \frac{dl \sin\phi}{r^2}$$

here $dl = dz$

$$d\bar{B} = \frac{\mu_0 i}{4\pi} \frac{dz \sin\phi}{r^2}$$

From Diagram:

$$\sin\phi = \sin(\pi - \phi)$$

$$d\bar{B} = \frac{\mu_0 i}{4\pi} \frac{dz}{r^2} \sin(\pi - \phi)$$

$$d\bar{B} = \frac{\mu_0 i \, dz}{4\pi r^2} \cdot \frac{d}{r}.$$

$$d\bar{B} = \frac{\mu_0 i \, d}{4\pi r^2 \cdot r} dz$$

From diagram $r^2 = (z^2 + d^2)$

$$\sqrt{r^2} = \sqrt{z^2 + d^2}$$

$$r = \sqrt{z^2 + d^2}$$

using in above equation.

$$d\bar{B} = \frac{\mu_0 i \, d \, . \, dz}{4\pi (z^2 + d^2)(z^2 + d^2)^{1/2}}$$

$$d\bar{B} = \frac{\mu_0 i \, d \, . \, dz}{4\pi (z^2 + d^2)^{3/2}}$$

Integrating on b/s.

$$\int d\bar{B} = \int_{-L/2}^{+L/2} \frac{\mu_0 i \, d \, dz}{4\pi (z^2 + d^2)^{3/2}}$$

$$B = \frac{\mu_0 i \, d}{4\pi} \int_{-L/2}^{+L/2} \frac{dz}{(z^2 + d^2)^{3/2}}$$

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Put $z = d\tan\theta$

$dz = d\sec^2\theta\, d\theta$.

So we get,

$$B = \frac{\mu_0 i\, d}{4\pi}\int_{-L/2}^{+L/2}\frac{d\sec^2\theta\, d\theta}{(d^2\tan^2\theta + d^2)^{3/2}}$$

$$B = \frac{\mu_0 i d}{4\pi}\int_{-L/2}^{+L/2}\frac{d\sec^2\theta\, d\theta}{d^3(1+\tan^2\theta)^{3/2}}$$

$$B = \frac{\mu_0 i d}{4\pi}\int_{-L/2}^{+L/2}\frac{d\,\cancel{\sec^2\theta}\, d\theta}{d^3\sec^3\theta}$$

$$B = \frac{\mu_0 i\, d.d}{4\pi}\int_{-L/2}^{+L/2}\frac{d\theta}{d^3\sec\theta}$$

$$B = \frac{\mu_0 i\, \cancel{d^2}}{4\pi\, d^{\cancel{3}}}\int_{-L/2}^{+L/2}\cos\theta\, d\theta$$

$$B = \frac{\mu_0 i}{4\pi d}\left[\sin\theta\right]_{-L/2}^{+L/2}$$

From diagram

$$\sin\theta = \frac{\text{Prep}}{\text{hyp}} = \frac{z}{r}$$

So,

$$B = \frac{\mu_0 i}{4\pi d}\left(\frac{z}{r}\right)$$

$$B = \frac{\mu_0 i}{4\pi d}\left(\frac{z}{\sqrt{z^2+d^2}}\right)_{-L/2}^{+L/2}$$

$$B = \frac{\mu_0 i}{4\pi d}\left[\frac{L/2}{\sqrt{(L/2)^2+d^2}} + \frac{L/2}{\sqrt{(L/2)^2+d^2}}\right]$$

$$B = \frac{\mu_0 i}{4\pi d}\left[\frac{L}{\sqrt{(L/2)^2+(d^2)}}\right]$$

using the approximation

$L >>>> d$

$d \longrightarrow 0$

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So,

$$B = \frac{\mu_0 i}{4\pi d}\left[\frac{L}{\sqrt{(L/2)^2 + (0)}}\right]$$

$$B = \frac{\mu_0 i}{4\pi d}\left[\frac{L}{(L/2)}\right]$$

$$B = \frac{\mu_0 i \cancel{L}}{{}_2\cancel{4}\pi d(\cancel{L}/\cancel{2})}$$

$$B = \frac{\mu_0 i}{2\pi d}$$

$$\boxed{B = \frac{\mu_0 i}{2\pi d}}$$

Source : HRK

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