Sinusoidal Regression Modeling Task

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Activity: Sine of Best Fit?

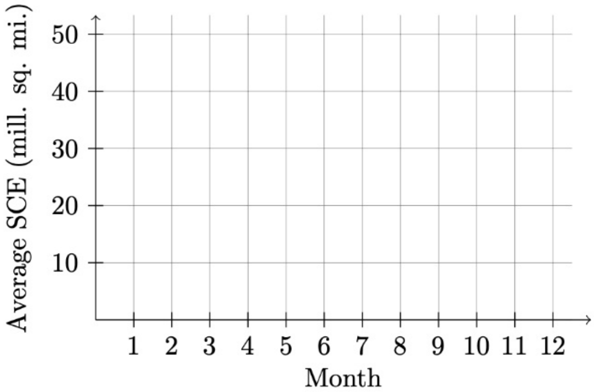
In the previous lesson, we encountered how sinusoidal functions can model periodic behavior like stock prices or even broccoli harvests. In this activity, we'll see how we can *create* such models.

Rutgers University, in conjunction with the US National Ice Center, tracks how much of the Northern Hemisphere is covered in snow at any given point in millions of square kilometers. This can be summarized by the annual snow cover extent (SCE), which has been recorded for the last 55 years. The table below shows the mean SCE for the Northern Hemisphere for the 55 years from 1966 to 2020. (Source: https://climate.rutgers.edu/snowcover/files/Robinson_snowdata2021.pdf)

Month	**Mean SCE**
January	47.1
February	46.0
March	40.4
April	30.5
May	19.1
June	9.4
July	3.9
August	3.0
September	5.4
October	18.6
November	34.3
December	43.7

As scientists learn more about climate change, the SCE data may provide evidence as to the speeding up or slowing down of certain climate trends.

1. Sketch a scatterplot of the data on the grid below, letting each month be represented by a number 1-12.



2. What kind of function do you think could be used to model the relationship between time and SCE?

3. Your calculator can be used to estimate a specific function that will model this relationship. First, you must put the data into *lists*. Press `stat` then `1:Edit...`.

4. We want to put the months into L1 and the SCE data into L2. If there is any existing data in L1 and L2, go to the top of the lists, highlight the list name (L1 or L2), then hit `clear` and `enter`.

5. Input the months into L1 and the SCE data into L2.

6. To view a scatterplot of the data, press `2nd` then `y=` (`stat plot`). On `1:Plot1`, press `enter` and turn it `On`. Select the scatterplot icon (the first one), make sure `Xlist:L1` and `Ylist:L2`. Then, press `zoom` `9:ZoomStat`.

7. Our goal now is to *construct* a sinusoidal function that can model this data. The function will be of the structure

$$f(x) = a\sin\left(b(x-c)\right) + d$$

8. Press `apps` and find `Transfrm`. Press any key to leave the application menu.

9. Go to `y=` and input the following into `Y1`:

```
A*sin(B(X-C))+D
```

Then, press `graph`.

0. Here are guidelines for using `Transfrm`:

- To adjust `A,B,C,` or `D`, you press the left and right arrows.
- To switch between `A,B,C,` and `D`, use the up and down arrows.
- If you want to input a specific value for any of `A,B,C,` or `D`, simply select the letter and then start typing your number.

1. Describe what effect changing each coefficient has on the graph of $f(x) = a\sin\left(b(x-c)\right) + d$.

- Changing a ...
- Changing b ...
- Changing c ...
- Changing d ...

12. Construct a function that you think *closely* models the given data. Report your values below.

$a =$ $b =$ $c =$ $d =$

It turns out that each and every one of these values can actually be approximated using data from the table!

13. How is your value of a related to the data in the table?

14. How is your value of d related to the data in the table?

15. How is your value of b related to the data in the table?

16. How is your value of c related to the data in the table?

Month	**Mean SCE**
1	47.1
2	46.0
3	40.4
4	30.5
5	19.1
6	9.4
7	3.9
8	3.0
9	5.4
10	18.6
11	34.3
12	43.7

17. Do you think that your function is the ***best possible*** approximation of the data?

You have previously learned about *lines of best fit.* More formally, these are called regression lines, and the process of finding them is called performing regression. It turns out that you can perform regression for *many kinds of functions besides linear ones*! In particular, your calculator can perform sinusoidal regression.

18. Press `stat`, then `CALC`, then scroll down and select `C:SinReg`. Input the following settings:

    `Iterations:3`

    `Xlist:L1`

    `Ylist:L2`

    `Store RegEq:Y2` (press `alphatrace2:Y2`)

    For `Period`, we have the option of inputting what we *think* the period should be. What should you put here, if anything?

The mathematics behind performing sinusoidal regression are beyond this class (they are reserved for future calculus courses), but *approximations* of regression functions can be created using the ideas from #13-16.

22. The previous SCE data was the 55-year average for 1966-2020. Below is the SCE for 2021.

Month	**2021 SCE**
January	46.8
February	46.1
March	38.6
April	28.8
May	16.2
June	6.2
July	2.8
August	2.5
September	5.6
October	18.1
November	35.4
December	44.5

23. Input the data into L3 and L4. Then, construct a scatterplot in your calculator so that you can see both data sets. (To *hide* the `Y=` functions without deleting them, go to `y=` and put the cursor on the equal sign (which will be shaded black) and press `enter`.) How is the 2021 data similar or different than the 1966-2020 averages?

24. Use the data to construct a model $g(x) = a \sin(b(x - c)) + d$.

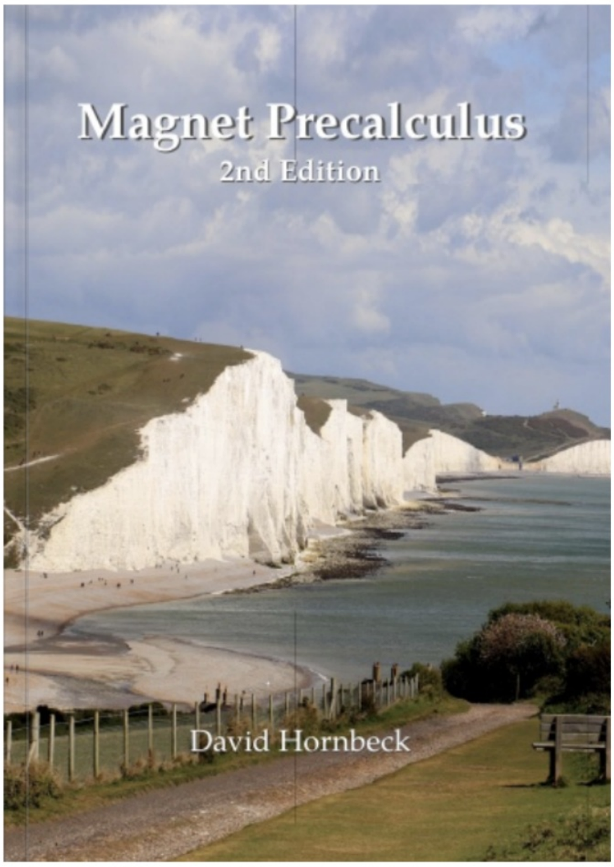
25. Perform sinusoidal regression to see how well you did!

26. Use both regression functions (for the 1966-2020 data and for the 2021 data) and predict the SCE in April 2024. Which estimate do you think is more reliable?

Summary

When two variables co-vary in a smooth and periodic manner, a sinusoidal function can often used to model the data accurately and make predictions. Using information like maxima, minima, and period, we can actually approximate these functions by hand. With a calculator, we can find the mathematically "best" regression equations.

Book raffle!



Questions?

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Feedback Form

Practice Question

In a blood pressure study, a participant's blood pressure was measured and automatically recorded every 0.05 seconds for 1 second. The table below displays the blood pressure, in millimeters of mercury (mmHg) recorded at each time.

Time (s)	Blood Pressure	Time (s)	Blood Pressure
0.05	116	0.55	53
0.1	120	0.6	60
0.15	116	0.65	71
0.2	109	0.7	88
0.25	98	0.75	98
0.3	85	0.8	110
0.35	70	0.85	116
0.4	60	0.9	119
0.45	53	0.95	115
0.5	50	1	109

(a) Find a function $f(x) = a\sin(b(x - c) + d$ that could reasonably model the data.

(b) Perform a sinusoidal regression to find a model for the data. Give it an estimate for the period.

(c) Perform a sinusoidal regression to find a model for the data *without* giving it an estimate for the period. How much does this change the model?