AP Precalculus:
The Teacher's Compendium

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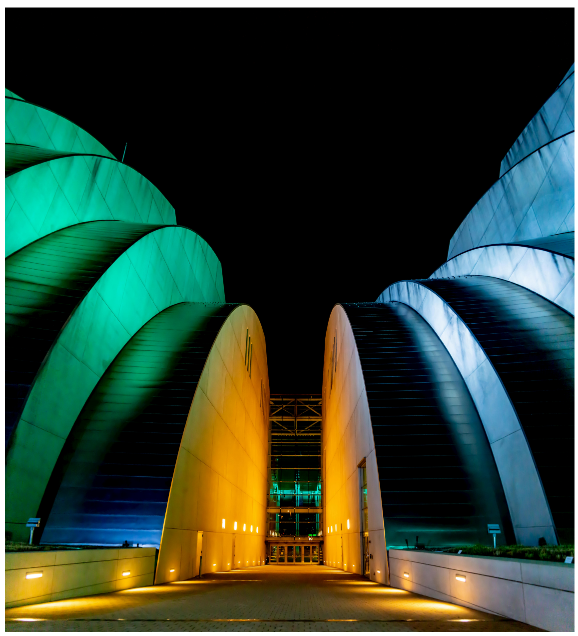

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AP Precalculus: The Teacher's Compendium

by David Hornbeck and Chuck Garner

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Preface

I recall being present for the College Board Forum at the AP Calculus Reading in the summer of 2017 in Kansas City. In that forum, Trevor Packer was asked by an audience member if there were plans to introduce an AP Multivariable Calculus. Trevor said that maybe if there was enough demand. He then asked those assembled, "Raise your hand if your school would be interested in such a course, or if your institution would likely give credit for such a course." I seem to remember that slightly more than half the hands raised. The person who asked the initial question said "It seems you have your demand." "It sure does," Trevor replied, and then he quipped, with a smile on his face, "too bad there are no plans to develop such a course." The crowd laughed, realizing that he was just leading us on. Then he said "But we are working on an AP Precalculus course which we plan to roll out in a few more years." The crowd was a little shocked, and more questions were lobbed at Mr. Packer. This was the first I had ever heard of AP Precalculus!

I've taught AP Calculus since 2002. I have created many materials for AP Calculus (a series of textbooks, an AP Exam preparation book, and an AP Calculus Problem Book) and used them with success in my classes. Others who have used my materials also report them as being useful and beneficial for students. When David Hornbeck applied for a position at my school in 2021, one of the things that stood out for me was that David also created his own materials for teaching AP Statistics at his previous school. Of course, we hired David immediately to teach Precalculus, AP Statistics, and a section of AP Calculus AB for which he used my materials. We also jointly sponsored our school's competitive Math Team. We spent many hours together during our common planning period, during Math Team meetings after school, at math conferences, and district professional development discussing ways to teach mathematics. I had never had a colleague who is as interested as I am in finding the best to approach every topic we teach, and in creating the materials for that approach. Our discussions resulted in making each of us better teachers.[1] Indeed, his creation of a Precalculus textbook (something I had never done in my seven years teaching that course) motivated me to create a textbook for my Discrete Mathematics course.

So when AP Precalculus was announced, and our school decided we should offer it, we knew David had to teach it, and he didn't think twice about creating his own materials. He started in the Summer of 2023. As we discussed his plan for the book sitting in a Kansas City restaurant (I was there for the AP Calculus reading and David was there for the AP Statistics reading), we realized that this could be a rich resource for any teacher of AP Precalculus. We could include tests, activities, lessons, and extra problems. We were determined to make this a useful book for teachers, and we determined that in order to do that, the lessons, activities, and problems should be used in class first.

Therefore, throughout the 2023-24 school year, David wrote this book by creating the lesson or activity for his AP Precalculus class each day and then reflecting upon the activity the next day. Throughout, we had regular discussions concerning almost every aspect of the approach to the material. Once a unit's worth of lessons was completed, I edited it, rewrote a few things, fixed typos, and wrote around 60% of

[1] I feel it is appropriate to mention another colleague, Julie Matthews, who was the best supporter of our work, the best sounding board, and the best person to question us when we needed it. Julie listened to many of the discussions David and I had, and she brought up ideas she had for teaching Geometry and Algebra II and how they could fit with how we are teaching. She even volunteered to change the way she teaches certain things so they align with what her students see in later courses. The urge to create materials manifested in Julie as well, and she has put together an Algebra II Problem Book. The three of us have been an unstoppable trio of mathematics teachers.

the multiple-choice problems (and solutions for those and some of the ones David wrote). Doing this gave me great insight into how clearly and cleverly David built up each topic so the student could grasp it more easily. I was so blown away by the quality of his materials, I petitioned our school administrator to allow us to teach all precalculus (AP and non-AP) using this book as the basis for, and the approach to, the curriculum. I found out recently that my petition was granted.

As you read the book, you may wonder why things look a little better than you are used to seeing in many kinds of teacher-generated materials. That is because instead of using Word or GoogleDocs, this book was written in LaTeX, the free document typesetting language. The figures were all drawn in Ti*k*Z, a LaTeX graphics package. LaTeX can be viewed as a markup language for documents with lots of mathematics, and we highly recommend using it for your mathematical documents.

I truly hope you find this book useful. I would encourage you to use the lessons straight out of the book with your students *at first*, particularly if you have never taught AP Precalculus before. I would then encourage you to tweak the materials in this book to align with your teaching practices and your students' needs, and begin to create your own materials. Only you know how your students learn, how quickly or slowly they grasp concepts, and what you have taught them. You should use materials that reflect these situations. Please use the ones in this book, and then modify them to suit your students' needs! This including writing your own tests and quizzes. While we provide sample tests and many good multiple-choice problems for both quizzes and tests, a test that you create in the style and substance of the AP exam is more appropriate than using someone else's test (including ours). To that end, we have included a short guide at the end of the book concerning how to write good problems.

I wish your students success in AP Precalculus!

CHUCK GARNER
CONYERS, GA, APRIL 2024

Introduction

In this book, you will find my lessons, activities, homework, reflections, and sample test items for an entire year of AP Precalculus. There are a few things to know about how the book is laid out and how these lessons were, or could be, implemented.

This book is ordered identically to (a) how College Board lays out the course in the Course and Exam Description and (b) how I taught the lessons. Each lesson corresponded to one 90-minute period, but most of those presented could be split into two 45- or 50-minute lessons. The vast majority of lessons have one of two structures: student-driven activities or teacher-led lessons. For the student-driven activities, I had students in groups of 3 or 4, and they would work through the activity for anywhere from 45 to 80 minutes. For teacher-led lessons, I would simply be at the board working through various examples. Any of the student-driven activities could easily be converted into a whole-class lesson, and teacher-led lessons could also be utilized as student-driven activities or practice days.

Many teachers decided to deviate from College Board's ordering of the material: placing Unit 3 first, rearranging Unit 3, or moving around individual topics. I chose to follow the course as designed by College Board for the first implementation of the course for the sole purpose of being able to utilize Progress Checks and to see how it went. Now, having taught the course, I completely stand by the original ordering. Though the lessons in this book could be taught in a different order, I would be very hesitant to change much, as I would fear students not making certain connections.

The bulk of the lessons are student-driven activities, which was a worthwhile departure from my own teaching style for 10 years. The activities try to thread the needle between completely open-ended tasks that students may give up on quickly and those insidious "activities" that are just lectures on paper. Throughout the course of the year, I saw students really engage with these activities, and the level of scaffolding seemed generally appropriate without sacrificing rigor. The activity structure does come at the expense of "practice" time, though, and this is where a teacher's discretion and knowledge of their own students' skills come into play.

In quite a few of the activities, there are links to Geogebra sketches. I am a firm believer in using animations to help mathematics come alive for students, so that they may truly *see* the mathematics, and I found that students responded well to them. These sketches are public, and I would encourage any interested teacher to try downloading them to look under the hood and see how they were made. Even something as simple as animating of the graphing of a function can transform how a student understands a concept, and such things aren't nearly as complicated in Geogebra as they may seem on first glance.

You may find that the book is short on "review" and "practice." This was not borne out of some philosophical reason, but rather just a shortage of time. My classes are on a modified block schedule, meaning I only saw them every other day. With disruptions – field trips, assemblies, etc. – and the timing of the AP exam, I really only had about 75 days with my students. When I saw a need for more skill practice, I would frequently assign Delta Math or write worksheets. For review, I would have my students work through the Progress Checks in AP Classroom (which I would print out, as I'm a firm believer in doing math on paper!). The sample pacing provided details when I would utilize these Progress Checks. If I had more class days, though, I would most certainly include more days for skill practice and review.

The homework assignments in this book were intended to give students 30 minutes to an hour of homework for each class period. The problems in these assignments are intended to mimic the style and rigor of AP exam questions, only without the multiple choice or free response format. By and large, the problems eschew any rote practice, but when this was needed, supplemental online assignments were created. The beginning of each class period was usually dedicated to going over any questions students had from these assignments. For my students, I actually posted the solutions to the homework online in their learning management system.

This book makes use of the 🖩 symbol at various times. When the 🖩 symbol is featured in the title of a lesson, it's meant to indicate that the lesson itself will rely heavily on a graphing calculator (or may even be a calculator-based investigation). The absence of this symbol does not, of course, mean that the calculator is prohibited or will not be used at all. In lessons in which the 🖩 symbol is missing, the symbol may again appear on individual questions within the lesson. The reflections at the end of each lesson should indicate the extent to which calculators should be or were used.

The book makes the assumption that students are working with a TI-84 (or TI-83) graphing calculator. This is partly selfish, as my own students had access to a class set of TI-84 calculators, and it certainly is the industry standard. That said, I also stuck with the TI-84 because including instructions for other common calculators – the NSpire, Numworks, various Casio models – would have ballooned the lessons to an unacceptable length. For students in my own class with other calculator models, I would simply provide assistance where I could and direct them to online manuals or videos.

You will find in this book a reflection at the end of each lesson. These reflections were written within a day of teaching or facilitating each lesson, and they're meant to provide insight into what went well, what didn't go so well, and what instructional techniques I may have used that aren't explicitly featured in the lesson. Each reflection also contains 5 sample multiple choice items. These items could be used on formative assessments like exit tickets or quizzes or on summative tests. Each and every item should be very similar to those that students might see on the AP exam.

Solutions to the activities, lessons, and problems are all included, and it is worth noting that, in most cases, multiple solutions may be appropriate. The solutions written here were meant to reflect either the most efficient or mathematically sound approach.

I would like to thank my co-author, Chuck, for his work in proofreading and editing this book and writing tremendous problems and solutions. Beyond his tireless and thorough work, I credit many of the conversations I had with him for the inspiration for many of these lessons. Had I never worked with Chuck, I never would have retaught myself LaTeX and definitely never would've thought to self-publish. For years now, he has been incredibly patient with me, answering my near constant questions about everything from precalculus and calculus to publishing and LaTeX.

I would additionally like to thank all of my students this year. They were the guinea pigs for these lessons, and their insights were both fascinating and invaluable. Without them, none of these lessons would exist!

I hope this book serves you and your students well.

David Hornbeck
Conyers, GA, April 2024