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Preface

This book was first born in January 2024 somewhat out of necessity. AP Precalculus[1], the course, was new in 2023-2024. No one had many resources. All anyone had to guide them as they prepared lessons and assessments was the Course and Exam Description (CED) for the course.

In the process of teaching the course and writing my own materials in 2023-24, I found myself poring through the CED, AP Classroom, and resources shared by others to the point that I came to know the course and, as a result, the exam, quite intimately. Though an AP Exam for AP Precalculus had not even been offered yet, the level of detail about the exam that is publicly available made writing practice materials more accessible than I thought previously imaginable. While other courses may require years of seeing released free response questions before similar high-quality questions can be written, the AP Precalculus free response questions are heavily scripted. The multiple choice section comes with clearly delineated breakdowns of content, skills, and presentation.

When I first thought about what I was going to do for AP exam review, I of course considered purchasing a set of review books from one of the major test prep companies. Then, I thought, if I know the material so well, have taught the course for as long as anyone else, and written lessons and assessments for my students all year, why not write one myself?

Thus, the book was born.

I wrote the first edition of this book between November of 2023 and February of 2024, constantly thinking of how I could weave the tremendous amount of content in this course into a few weeks' worth of sweeping lessons. The goal of this book was for a teacher to be able to work with their students through the eight lessons, have their students practice with multiple choice and free response questions similar to those that will be on the AP exam, and ultimately help students "ace" the AP Precalculus exam.

For the 2024-25 school year, I decided to add some additional material to this book informed by both my own students' performance on and the released scoring guidelines for the inaugural 2024 AP Precalculus exam. Specifically, I added a Digital Exam FAQs page to answer some of the most pressing questions students may have; Lesson 8 and more problems in Appendix D: Skill Practice to help students navigate the scripted portions of the FRQs; and a new appendix, Appendix D: Additional FRQ #4s, to give students more practice with what many of them find the most challenging FRQ. I also made minor improvements and fixed a handful of typos.

I would like to thank my wife, MaryBeth, for her unwavering support, and my own AP Precalculus students last year. They were the real pioneers, and their experiences with this course are what informed so much of what has gone into both the original book and this second edition. Without them, this book would be meaningless to me. I would also like to thank Chasen Smith for his proofreading this book. His eye for detail and precision of vocabulary are unmatched, and I learn something about mathematics every time I talk with him.

David Hornbeck
October 2024

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Introduction

There are a few things to know about this book and how it's intended to be used.

Structure of the Book
The lessons are based on 90-minute class periods, but they could easily be split up into 45- or 50-minute chunks as needed. In my own classroom, I plan to be at the board, working through all of the examples with students as they record the solutions in their own notes.

The structure of the eight lessons is meant to spiral students through the course material in a way similar to what they can expect on the AP exam. The exam won't explicitly cover Unit 1 to Unit 3 in order, and so neither should a review book. That said, organizing into convenient "big ideas" was more difficult than I thought. The book starts with arguably the biggest idea in the course - rates of change - and from there attempts to mix in material from all different units into some of the major concepts and skills covered in the course (with an exception of Lesson 5, which explicitly covers Unit 3).

The number eight for the lessons in this book was born out of my own scheduling constraints. I see my students either twice or thrice a week for 90 minutes, so a month of review corresponds to only 10 or 11 class periods. Some of these days may be used for giving partial mock exams, and others will see students out for field trips, assemblies, or other AP exams. I ended up settling on eight lessons so as to give myself some flexibility.

Mini-AP Exams
Each lesson comes with a "mini AP-exam." Unlike the mock exams in this book, these are not explicitly designed to satisfy any content breakdowns. Some questions may even appear somewhat out of place with regards to the lesson. This was an inevitable consequence of trying to cover so much material per lesson. Rather than limit the mini-exams to the exact types of problems covered in the lessons, though, I decided to keep certain questions that would push students or seem not entirely related to the lesson. The AP exam will throw curveballs at students, so I wanted this book to do the same.

Solutions
The mini-exams lead to an important note about this book: there is a separate solutions manual. Ideally, a teacher using this book could use the mini- or mock exams as actual classroom assessments (what gets students to study more than giving a test?). Providing solutions in this book would make this impossible, so I decided to include all of the solutions to the mini-exams, mock exams, and skill practice in a separate manual.

To discourage students from buying the solutions themselves, I have made the publicly searchable solutions manual prohibitively expensive. However, if a teacher were to buy 10 or more copies of this book, there will be a private link where the solutions manual will be sold for $5, less than the cost to takes to print the book. Simply e-mail me a receipt at dhornbeck@rockdale.k12.ga.us and I will send this private link.

Mock Exams
Mock exams are a prized commodity in the AP world, but their quality can vary wildly. This book comes with two of them, but why should you trust them?

I cannot claim these exams are perfect by any stretch of the imagination, but what I can assure you of is that they have been designed to meet all of the specifications of the AP exam that have been released publicly. Each exam was crafted to satisfy the percentages of function types, Skills, and calculator skills and usage. The presentation types - graphical, analytical, tabular, and verbal - were varied similarly to

what can be expected on the exam. The free response questions were crafted carefully and thoroughly to mimic as closely as possible what students should expect to see on the exam.

It was my intent that these mock exams be ever so slightly more difficult than what I anticipate the actual AP exam will be. In my experience, I have found that over-preparing students and possibly giving them a conservative impression of what they will score is far more beneficial than its optimistic counterpart. The cut scores you will find in the solutions manual are mere approximations, but I hope that they end up being a bit higher than those on the actual exam.

Appendices
In addition to the eight lessons, I decided to add in appendices that I thought would be useful for my students. In order to succeed on the AP exam, students will need proficiency with certain skills and a deep knowledge of numerous function families. Students will also need to know certain trigonometric identities, and it's important that they know exactly what identities they'll be responsible for. From the 2024 AP exam, it was also apparent that FRQ #4 provided an extra challenge, so 10 more sample #4s have been added as a new appendix. It is my hope that these appendices could be used by teachers as additional review lessons or supplementary homework/practice. It is worth noting that answers to both Appendix D and Appendix E are available in the solutions manual.

Calculators
This book makes the assumption that students are working with a TI-84 graphing calculator. This was both a selfish decision, as my own students have access to a class set of these calculators, but also a practical one. If I included instructions for and solutions based on TI-Nspires, various Casio models, and the Numworks graphing calculator, the book would become substantially longer, more expensive, and even possibly give the impression that the calculator is of the utmost importance. For students who elect to use the Desmos app in Bluebook on the AP exam, there are ample video series on Youtube to guide them on how to best use Desmos.

Digital Exam FAQs

When you take the AP Precalculus this year, the exam will be *digital*. Many details can be found on the College Board website, and this should be your resource for researching any questions you may have about the digital exam format, but we'll include here some of the most frequently asked questions regarding the digital exam.

What will the digital exam look like?

The digital exam will occur in Bluebook, the same application that you may have taken the PSAT or SAT on. If you worked any items in AP Classroom this year, they should have resembled what the questions will look like in Bluebook.

I'm used to underlining information and crossing out answers on paper exams. Will I be able to do this in Bluebook?

Yes! Bluebook contains tools to highlight information and to cross out multiple choice answers, as well as to mark questions for later review, read only 1-2 lines at a time, and more.

Will scratch paper be allowed?

Yes! You will be provided with scratch paper.

How will I answer FRQs digitally?

You won't! The free response questions will be displayed in Bluebook, but you will receive a separate answer document on which you will record your solutions to the FRQs.

Will there be an online calculator in Bluebook?

Yes! Bluebook will have an app-based version of Desmos available on the AP exam. Previews should have been made (or will be made, depending on when you're reading this) by the spring of 2025 so that you can see what the Desmos graphing calculator will look like within Bluebook.

Does this mean I can't use my handheld graphing calculator?

No! You can still use your handheld graphing calculator provided it is on the approved calculator list (which you can find on the College Board website). If you are familiar with Desmos or wish to become familiar, this is great, but you should use the calculator that you are most familiar with and used all year on the AP exam.

LESSON 1

Rates of Change

AP Precalculus is, among other things, a course about functions and the relationships between their inputs and outputs. From linear all the way through polar, functions describe how inputs and outputs change together. This idea is called *covariation*.

Throughout the course (and this book), the relationships between inputs and outputs of functions were expressed in a variety of ways: analytically, graphically, verbally, and tabularly. As you work through this book, it will be vital that you're comfortable with the varying representations of the concepts.

One of the key concepts when examining how inputs and outputs change together is **rate of change**, which describes how the outputs are changing as the inputs change. Given a function $y = f(x)$ and any two input-output pairs $(x_1, f(x_1))$ and $(x_2, f(x_2))$, the change in the outputs is $f(x_2) - f(x_1)$. The rate of change over the **interval** $x_1 \le x \le x_2$ can therefore be approximated by the change in outputs divided by the change in inputs or

$$\text{Rate of change} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

This quotient is called the **average rate of change**. If this equation seems familiar, it is because it is the **slope** of the line going through the points $(x_1, f(x_1))$ and $(x_2, f(x_2))$. The slope of this line, called a **secant line** for a function, is the graphical interpretation of the average rate of change. There are, of course, other representations, as shown in Figure 1.1. When the rate of change of a function is positive, we say the function is **increasing**; when the rate of change is negative, we say the function is **decreasing**.

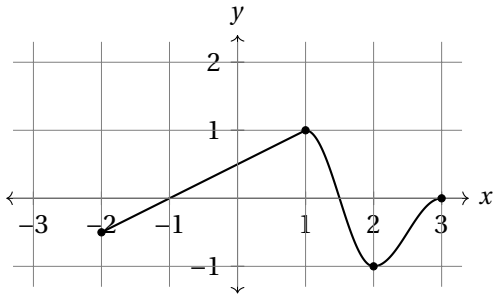
Example 1.1: Rates of Change Graphically

Let $f(x)$ be the function shown in the graphed in the xy-plane below for $-2 \le x \le 3$.

(a) On what interval/s is f increasing?

(b) Find the average rate of change of f on the interval $-2 \le x \le 3$.

(c) On which interval, $[-2, 1]$, $[1, 2]$, or $[2, 3]$, is the average rate of change of f the greatest?



★ AP Tip! ★

When interpreting an average rate of change in context, use a script similar to the following:

From x_1 to x_2, the outputs increased/decreased by $|ARC|$ units (of the output in context) for every additional unit (of the input in context).

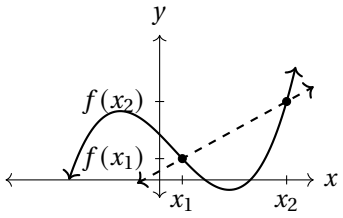
<table>
<tr><th colspan="2">Average Rate of Change (ARC)</th></tr>
<tr><td>Analytical</td><td>$\dfrac{\Delta f}{\Delta x} = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$</td></tr>
<tr><td>Graphical</td><td>The slope of the secant line
 
</td></tr>
<tr><td>Tabular</td><td><table>
<tr><th>x</th><th>$f(x)$</th><th>Difference</th></tr>
<tr><td>x_1</td><td>$f(x_1)$</td><td></td></tr>
<tr><td>x_2</td><td>$f(x_2)$</td><td>$f(x_2) - f(x_1)$</td></tr>
</table>
The ARC is the ratio of the difference in outputs to the length of the interval, $x_2 - x_1$.</td></tr>
<tr><td>Verbal</td><td>From x_1 to x_2, the outputs changed by an average of ARC for each increase of 1 in the inputs.</td></tr>
</table>

Figure 1.1: ARC representations

Solution

(a) f is increasing when the rate of change is positive, which occurs for $-2 \le x \le 1$ and $2 \le x \le 3$.

(b) $\frac{f(3)-f(-2)}{3-(-2)} = \frac{0-\left(-\frac{1}{2}\right)}{5} = \frac{1}{10}$

(c) The average rates of change are

$$\frac{f(1)-f(-2)}{1-(-2)} = \frac{1-\left(-\frac{1}{2}\right)}{3} = \frac{1}{2} \qquad \frac{f(2)-f(1)}{2-1} = \frac{-1-1}{1} = -2 \qquad \frac{f(3)-f(2)}{3-2} = \frac{0-(-1)}{1} = 1$$

The highest average rate of change of f is 1, which occurs on the interval $[2,3]$.

Rates of change enabled us to find **extrema**, which we identified as **local** or **global** **maxima** or **minima**.

Fact 1.A: Finding Extrema

For a continuous function f, a local/relative maximum will occur

- when the function switches from increasing (positive ROC) to decreasing (negative ROC) or
- at the included endpoint with a restricted domain

Don't forget endpoints!!

Similarly, a local/relative minimum will occur

- when the function switches from decreasing (negative ROC) to increasing (positive ROC) or
- at the included endpoint with a restricted domain

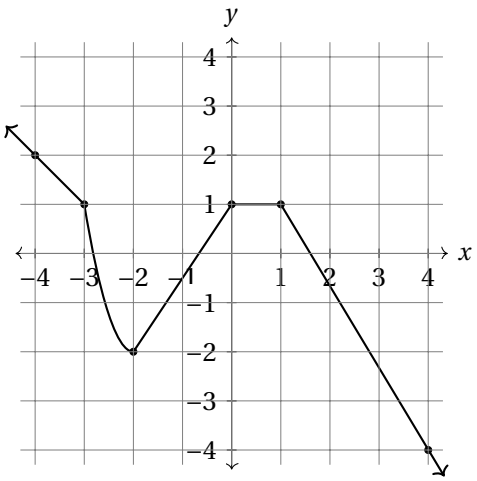
A global or absolute maximum/minimum occurs at the local maximum/minimum with the highest/lowest value among the maxima/minima, respectively.

Mini-AP Exam 1

Questions that allow a calculator will have the (🖩) symbol.

Section I: Multiple Choice

1. The graph of the function f is shown. On which of the following intervals is the average rate of change of f the least?



(A) $[-4,-3]$ (B) $[-3,-2]$ (C) $[0,1]$ (D) $[1,4]$

2. The function $g(x)$ is a quadratic. Values of g are given in the table below.

x	1	4	7
$g(x)$	−5	1	10

What is $g(10)$?
(A) 20 (B) 22 (C) 28 (D) 34

3. Which of the following must be true about the polynomial function $f(x) = -3x(x-2)(x+1)$?

(A) f has a global maximum or minimum when $x = 1$.
(B) f has a global maximum or minimum when $x = c$ for some c with $0 < c < 2$.
(C) f has a local maximum or minimum when $x = 1$.
(D) f has a local maximum or minimum when $x = c$ for some c with $0 < c < 2$.

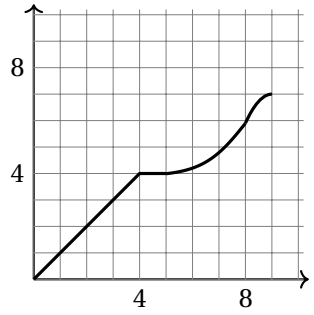
4. (🖩) Let $f(x) = \frac{1}{2}x^4 - 2x^3 + \frac{3}{2}x^2 + 2x - 2$ and let $g(x) = f(x)$ for $-1 \le x \le 3$. How many total local maxima and minima does the graph of g have?
(A) Two (B) Three (C) Four (D) Five

5. The function h is given by $h(x) = -2(x-1)^2 + 3$, defined for all real numbers. Which of the following is true?

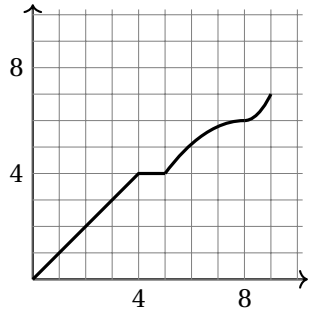
(A) The graph of h is concave down only for $x < 1$.
(B) The graph of h is concave up only for $x < 1$.
(C) The graph of h is concave down for all x.
(D) The graph of h is concave up for all x.

6. A proofreader recorded their number of articles that they read and edited over the course of a day. For the first four hours of the day, the proofreader read 1 article per hour. After this, they took a break for an hour. After lunch, they slowly increased their speed for 3 hours until finally slowing down in the last hour of the day. Which of the following could be a graph of $P(t)$, the number of articles P read t hours after arriving at work, where $t = 1$ would represent 1 hour after starting work?

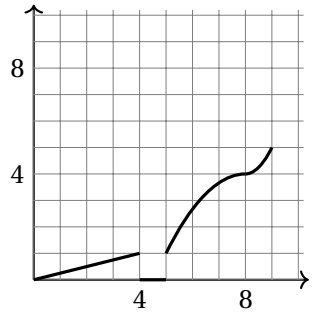
(A)



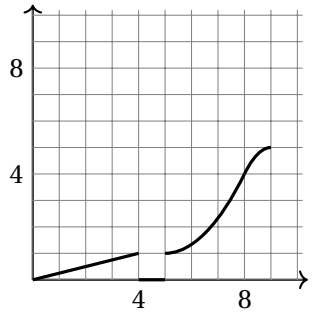
(B)



(C)



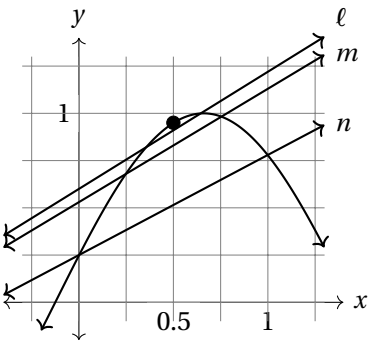
(D)



7. A swimmer wanted to practice holding their breath underwater for as long as possible, so they began cardiovascular training. They would test how long they could hold their breath underwater once every day. One week after beginning training, they could hold their breath for 4.5 minutes, and two weeks after beginning training, they could hold their breath for 5.9 minutes. The swimmer noticed that their time of holding their breath could be modeled by a linear function of the days since beginning training. How much improvement would the swimmer expect to make, in minutes, from day 14 to day 24 after beginning training?

(A) 0.2 minutes (B) 1.4 minutes (C) 2 minutes (D) 7 minutes

8. The graph of a function f is shown below, along with lines ℓ, m, and n. Line ℓ has a slope of 0.62, line m has a slope of 0.6, and line n has a slope of 0.53.



If the rate of change of f at $x = 0.5$ is r, which of the following is most likely to be true?

(A) $r < 0.53$ (B) $0.53 < r < 0.6$ (C) $0.6 < r < 0.62$ (D) $r > 0.62$