

Unit 3 Lessons Learned in AP Precalculus

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We're going to discuss some pedagogical tips related to Unit 3: Trigonometric Functions of the AP Precalculus Course and Exam Description.

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Strategy 1: Don't let students say "degrees."

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Strategy 1: Don't let students say "degrees."

Strategy 2: Use an applet and non-unit circle angles to develop radian sense.

<https://www.geogebra.org/m/hkeyfgj3>

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Strategy 1: Don't let students say "degrees."

Strategy 2: Use an applet and non-unit circle angles to develop radian sense.

<https://www.geogebra.org/m/hkeyfgj3>

HW Problem: Estimate the following.

(a) $\sin 3.4$ (b) $\tan 1.5$ (c) $\cos 0.05$ (d) $\cos \frac{12\pi}{23}$ (e) $\tan 1$

Where do the trig functions come from?

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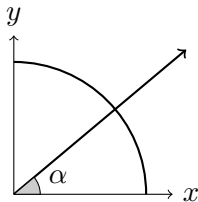
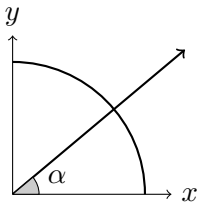
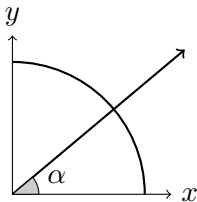
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But what do they represent graphically... and why are they named that way? As a matter of fact... where do the names “sine,” “cosine,” and “tangent” come from?¹



¹This was shown at a 2022 GMC presentation by Dennis Wilson!

Geometric Interpretations

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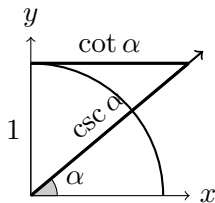
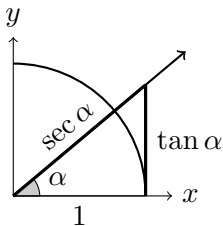
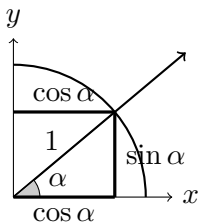
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A Helpful Analogy

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To motivate the unit circle, think of *unit price*.

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- Prepares students for polar coordinates immediately
- Prepares students for other applications of unit entities (unit vectors in calculus & physics)

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We all likely teach that the period of $f(x) = a \sin(b(x + c)) + d$ is $\frac{2\pi}{|b|}$. But what can be equally useful – though students often need it spelled out for them – is the alternative formula...

Period & b

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Feedback

We all likely teach that the period of $f(x) = a \sin(b(x + c)) + d$ is $\frac{2\pi}{|b|}$. But what can be equally useful – though students often need it spelled out for them – is the alternative formula...

$$|b| = \frac{2\pi}{\text{Period}}$$

Example: Find the period of

$$h(t) = 2.4 \cos\left(\frac{2\pi}{365}(x - 90)\right) + 0.3.$$

Speaking of the formula...

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The CED defines the parent sinusoidal functions using $b(x + c)$, *not* $b(x - c)$.

How do we get our students to handle this?

Speaking of the formula...

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The CED defines the parent sinusoidal functions using $b(x + c)$, *not* $b(x - c)$.

How do we get our students to handle this?

- Inputs are unintuitive because function has to *compensate*

Speaking of the formula...

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The CED defines the parent sinusoidal functions using $b(x + c)$, *not* $b(x - c)$.

How do we get our students to handle this?

- Inputs are unintuitive because function has to *compensate*
- Paper function example

Speaking of the formula...

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The CED defines the parent sinusoidal functions using $b(x + c)$, *not* $b(x - c)$.

How do we get our students to handle this?

- Inputs are unintuitive because function has to *compensate*
- Paper function example
- Show $(x + 2)^2$ v. x^2

Applets

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Two helpful applets for helping students compute a, b, c, d given a set of 5 points in the structure of FRQ #3.

- From points/a table:

<https://www.geogebra.org/m/jqkcxpse>

- From a graph:

<https://www.geogebra.org/m/jtb4fyd7>

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Interpreting $\tan \theta$ as the slope of the terminal ray of θ in standard position has a number of benefits, namely, understanding...

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Interpreting $\tan \theta$ as the slope of the terminal ray of θ in standard position has a number of benefits, namely, understanding...

- when $\tan \theta$ is undefined

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Interpreting $\tan \theta$ as the slope of the terminal ray of θ in standard position has a number of benefits, namely, understanding...

- when $\tan \theta$ is undefined
- when $\tan \theta = 0$

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Interpreting $\tan \theta$ as the slope of the terminal ray of θ in standard position has a number of benefits, namely, understanding...

- when $\tan \theta$ is undefined
- when $\tan \theta = 0$
- when $\tan \theta$ is positive or negative

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Interpreting $\tan \theta$ as the slope of the terminal ray of θ in standard position has a number of benefits, namely, understanding...

- when $\tan \theta$ is undefined
- when $\tan \theta = 0$
- when $\tan \theta$ is positive or negative
- the graph of $y = \tan \theta$

Pythagorean Identities Geometrically

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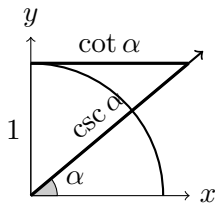
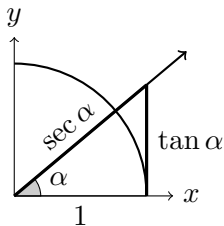
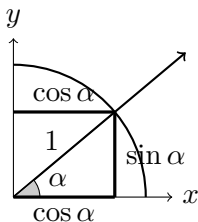
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The most important skill – beyond simply knowing identities – is diagnosing when to use them. What I’ve generally found is

	Before	Now
Day 1	Few identities, practice	Teach <i>all</i> identities
Day 2	Few identities, practice	Practice
Day 3	Few identities, practice	Practice
⋮	⋮	⋮

The “now” might take a couple days longer, but it’s worth it.

My 3.12, 3.14-3.15 Lessons from 2024

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<https://drive.google.com/file/d/1CnhPcNibN240XqfgMRATs2wnXNsDLGDT/view?usp=sharing>

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For graphs of polar functions, emphasize the graphs of the corresponding functions in the Cartesian plane.

- Graph non-trigonometric functions – great for rational functions
- For graphing polar functions, have them sketch a graph in the Cartesian planes.

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Would YOU like to practice writing AP items?

Would YOU like to contribute to math education in Georgia?

Would YOU be willing to sacrifice a couple hours' worth of your time?

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Would YOU like to practice writing AP items?

Would YOU like to contribute to math education in Georgia?

Would YOU be willing to sacrifice a couple hours' worth of your time?

Then do we have the opportunity for you!

Raffle

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Book Giveaway!

Thank you!

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Please fill out the feedback form at

<https://forms.gle/i8CQYqdBBiJM8zFs8>

or scan the QR code below.



Feel free to e-mail me at david.hornbeck@gcpsk12.org.