Writing Rigorous and Thought-Provoking Assessments

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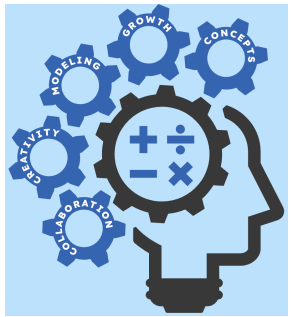

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- The assessment assumes a certain pace.
- Distractors on MC items should be based on reasonable incorrect student work.

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- AP Calculus AB Exam
  - Passing score (3) projected around 40%
  - B in college course (4) projected around 50%

Personal Philosophy

"If you aren't in over your head, how do you know how tall you are?" *T.S. Eliot*

If strong students are earning 100%s, then I only know that they're capable of *at least* the content on my exam - but I don't know their limits.

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- "Rigorous items are great, but I don't know where to find them."
  - Let's talk about that.

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- Spiraling content (question requires or connects to prior knowledge)
- Designing questions where process of elimination can't be easily used

Resources

- My own Google folder: `https://shorturl.at/01dfv`
  - So You Want to Write a Test? (AP Precalculus specific, but many ideas transfer to any course)
  - 27 sample tests from Geometry, Algebra II, Precalculus, and AP Precalculus
- Art of Problem Solving: Alcumus (free)
  - artofproblemsolving.com $\rightarrow$ Resources $\rightarrow$ Alcumus
- Released practice SATs (free)

Disclaimer

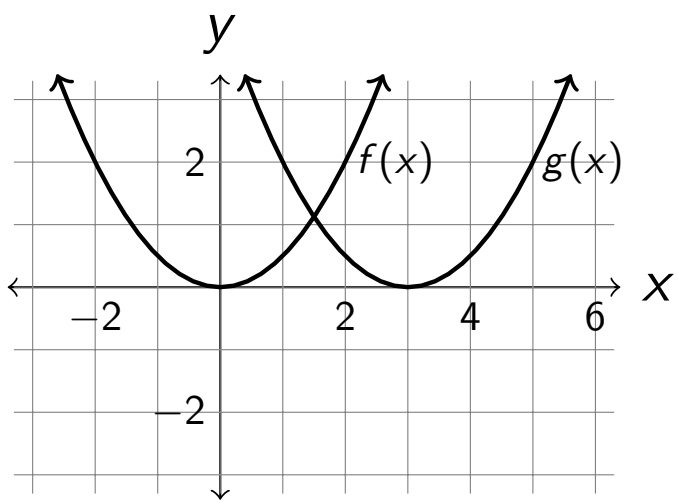
None of the sample questions to follow are "bad" questions! This is simply a demonstration of how questions with low cognitive loads can be transformed into more rigorous questions.

Sample Question 1: Algebra C&C

Learning Objective: A.FGR.7.2 Identify the effect on the graph generated by a quadratic function when replacing $f(x)$ with $f(x)+k$, $kf(x)$, $f(kx)$, and $f(x+k)$ for specific values of k (both positive and negative); find the value of k given the graphs

1. The functions f and g are graphed below. Which is an expression for $g(x)$?

(A) $g(x) = f(x+3)$
(B) $g(x) = f(x-3)$
(C) $g(x) = f(x)+3$
(D) $g(x) = f(x)-3$



Modified Examples of Sample Question 1

1. The graph of the function f contains the input-output pairs $(-2, 5)$ and $(6, -3)$. If the graph of the function g contains the input-output pairs $(1, 5)$ and $(9, -3)$, which of the following could be an expression for g in terms of f?

(A) $g(x) = f(x + 3)$

(B) $g(x) = f(x - 3)$

(C) $g(x) = f(x) + 3$

(D) $g(x) = f(x) - 3$

Modified Examples of Sample Question 1

1. The graph of the function f attains a maximum of 8 that occurs when $x = 2$. If g is the function defined by $g(x) = f(x - 3)$, which of the following is true?

(A) The graph of g attains a maximum of 5 when $x = 2$.

(B) The graph of g attains a maximum of 11 when $x = 2$.

(C) The graph of g attains a maximum of 8 when $x = -1$.

(D) The graph of g attains a maximum of 8 when $x = 5$.

Modified Examples of Sample Question 1

Let f be a function. Values of f for selected values of x are provided in the table.

x	-1	2	4
$f(x)$	3	0	5

If g is defined by $g(x) = f(x-3)$, which of the following tables could g satisfy?

(A)

x	-4	-1	1
$f(x)$	3	0	5

(B)

x	-1	2	4
$f(x)$	6	3	8

(C)

x	-1	2	4
$f(x)$	0	-3	2

(D)

x	2	5	7
$f(x)$	3	0	5

Sample Question 2: Geometry C&C

Learning Objective: G.GSR.6.3 Use trigonometric ratios and the Pythagorean Theorem to solve for sides and angles of right triangles in applied problems.

2. A 10-foot ladder leans against a house. The ladder makes an angle of 72° with the ground. How high on the side of house does the top of the ladder touch?

(A) 30.77 feet

(B) 9.51 feet

(C) 5.32 feet

(D) 3.09 feet

Modified Examples of Sample Question 2

2. A 10-foot climbing ramp goes from the ground to the top of a playground structure. The ramp makes an angle of 40° with the ground. Which of the following is an expression for the distance from the ground to the top of the ramp?

(A) $10 \sin(40^\circ)$

(B) $10 \sin(50^\circ)$

(C) $10 \cos(40^\circ)$

(D) $10 \tan(50^\circ)$

Modified Examples of Sample Question 2

2. A 10-foot climbing ramp goes from the ground to the top of a playground structure. The ramp makes an angle of 40° with the ground. Another ramp, also 10 feet long, ends at a point 1 foot lower off the ground than the other ramp. How does the angle that the second ramp makes with the ground compare to the angle that the first ramp makes with the ground?

(A) The angle that the second ramp forms with the ground is 7.13° larger.

(B) The angle that the second ramp forms with the ground is 8.24° larger.

(C) The angle that the second ramp forms with the ground is 7.13° smaller.

(D) The angle that the second ramp forms with the ground is 8.24° smaller.

Modified Examples of Sample Question 2

2. A 10-foot climbing ramp goes from the ground to the top of a playground structure. The top of the playground structure is $10\sin(40^\circ)$ feet from the ground. Which of the following is an expression for the slope of the ramp, in vertical feet per 1 foot along the ground?

(A) $\tan(40^\circ)$

(B) $10\tan(40^\circ)$

(C) $\frac{\cos(40^\circ)}{\sin(40^\circ)}$

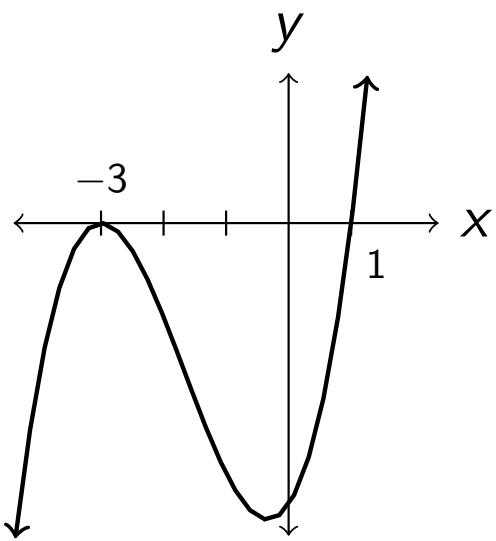
(D) 10

Sample Question 3: Advanced Algebra C&C

Learning Objective: AA.FGR.5.9 Identify zeros of polynomial functions using technology or pre-factored polynomials and use the zeros to construct a graph of the function defined by the polynomial function. Analyze key features of these polynomial functions.

3. Which of the following could be an expression for the function graphed below?

(A) $f(x) = (x-1)(x+3)$
(B) $f(x) = (x-1)(x+3)^2$
(C) $f(x) = (x+1)(x-3)$
(D) $f(x) = (x+1)(x-3)^2$



Modified Examples of Sample Question 3

3. Let f be the function defined by $f(x) = (x-1)(x+3)^2$ for all real x. Suppose f obtains a minimum value at $x = k$. Which of the following is true?

(A) $k < -3$

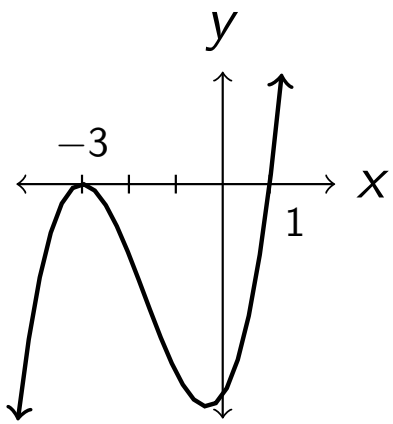
(B) $-3 < k < 1$

(C) $1 < k < 3$

(D) $k > 3$

Modified Examples of Sample Question 3

3. The polynomial function f is graphed below.



If $f(x) = (x-1)^a(x+3)^b$, where a and b are integers, then which of the following must be true?

(A) a and b are both odd.

(B) a is odd and b is even.

(C) a is even and b is odd.

(D) a and b are both even.

Modified Examples of Sample Question 3

3. The function g is defined by $g(x) = x(x-a)(x-b)^2(x-c)^3$, where $0 < a < b < c$. How many total extrema does the graph of g attain?

(A) 3

(B) 4

(C) 5

(D) 6

Sample Question 4: Precalculus

Learning Objective: PC.FGR.2.6 Represent the behavior of a rational function using limit notation for vertical and horizontal asymptotes and end behavior.

4. The graph of the function f has a horizontal asymptote of $y = 3$. Which of the following describes this using limit notation?

(A) $\lim_{x \to 3} f(x) = \infty$

(B) $\lim_{x \to \infty} f(x) = 3$

(C) $\lim_{x \to 3} f(x) = -\infty$

(D) $f(x) = \lim_{x \to \infty} 3$

Modified Examples of Sample Question 4

4. For a rational function f, it is known that $\lim_{x \to 3^-} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = 4$. Which of the following is true?

(A) The graph of f has a vertical asymptote of $x = 3$.

(B) The graph of f has a vertical asymptote of $x = 4$.

(C) The graph of f has a hole when $x = 3$.

(D) The graph of f has a hole when $x = 4$.

Modified Examples of Sample Question 4

4. Let f be the function $f(x) = \frac{1+x^2}{4x^2-3}$. Which of the following describes the outputs of f as the inputs increase without bound?

(A) The outputs will decrease without bound.

(B) The outputs will get arbitrarily close to 0.

(C) The outputs will get arbitrarily close to $\frac{1}{4}$.

(D) The outputs will increase without bound.

Modified Examples of Sample Question 4

4. The function f is defined by $f(x) = \frac{ax+2}{x+b}$. If $\lim_{x \to 3^+} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = 4$, then find $a + b$.

(A) -1

(B) 1

(C) 3

(D) 7

Using backwards design

When designing a test, it helps to lay out a structure first.

- What standard/topic will each question cover?
- How many questions will be easy? Medium? Challenging?
- What representations will you use?

I like to lay out a skeleton with a description of every question before I begin writing. This will include the standard/topic, representation, and a short descriptor for each item.

Thank you!

My Google Folder

https://tinyurl.com/HornbeckRigor

Session Evaluation Form

http://bit.ly/2024GMCsessions

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