AM, GM, Sequences & Series Introduction

An **arithmetic sequence** is one in which each consecutive terms share a common difference, i.e.

$$a, a + d, a + 2d, a + 3d, a + 4d, a + 5d, a + 6d, \ldots \qquad (\bigstar)$$

The terms in an arithmetic sequence satisfy an interesting property.

1. Pick any term in the arithmetic sequence (★). How is it related to the term immediately before it and the term immediately after it?

It turns out, this has a name: for any two numbers x and y, the quantity $\frac{x+y}{2}$ is the ________________________.

$$\text{GM} = \sqrt{xy}$$

More generally, for n numbers $a_1, \ldots, a_n$, the geometric mean is

$$\text{GM} = \sqrt[n]{a_1 a_2 \cdots a_n}$$

A **geometric sequence** is one in which each term is a constant multiple of the previous term, i.e.

$$a, ar, ar^2, ar^3, ar^4, ar^5, \ldots ar^n, \ldots \qquad (\Diamond)$$

The terms in a geometric sequence satisfy an interesting property.

2. Pick any term in the geometric sequence (◊). How is it related to the *product* of the term immediately before it and the term immediately after it?

It turns out, this has a name: for any two numbers x and y, the quantity $\sqrt{xy}$ is the ________________________.

$$\text{GM} = \sqrt{xy}$$

3. (Competition Example) A geometric progression has 5 terms, two of which are known.

$$g_n = \{3, g_2, g_3, g_4, 27$$

Find g_4.

Solutions