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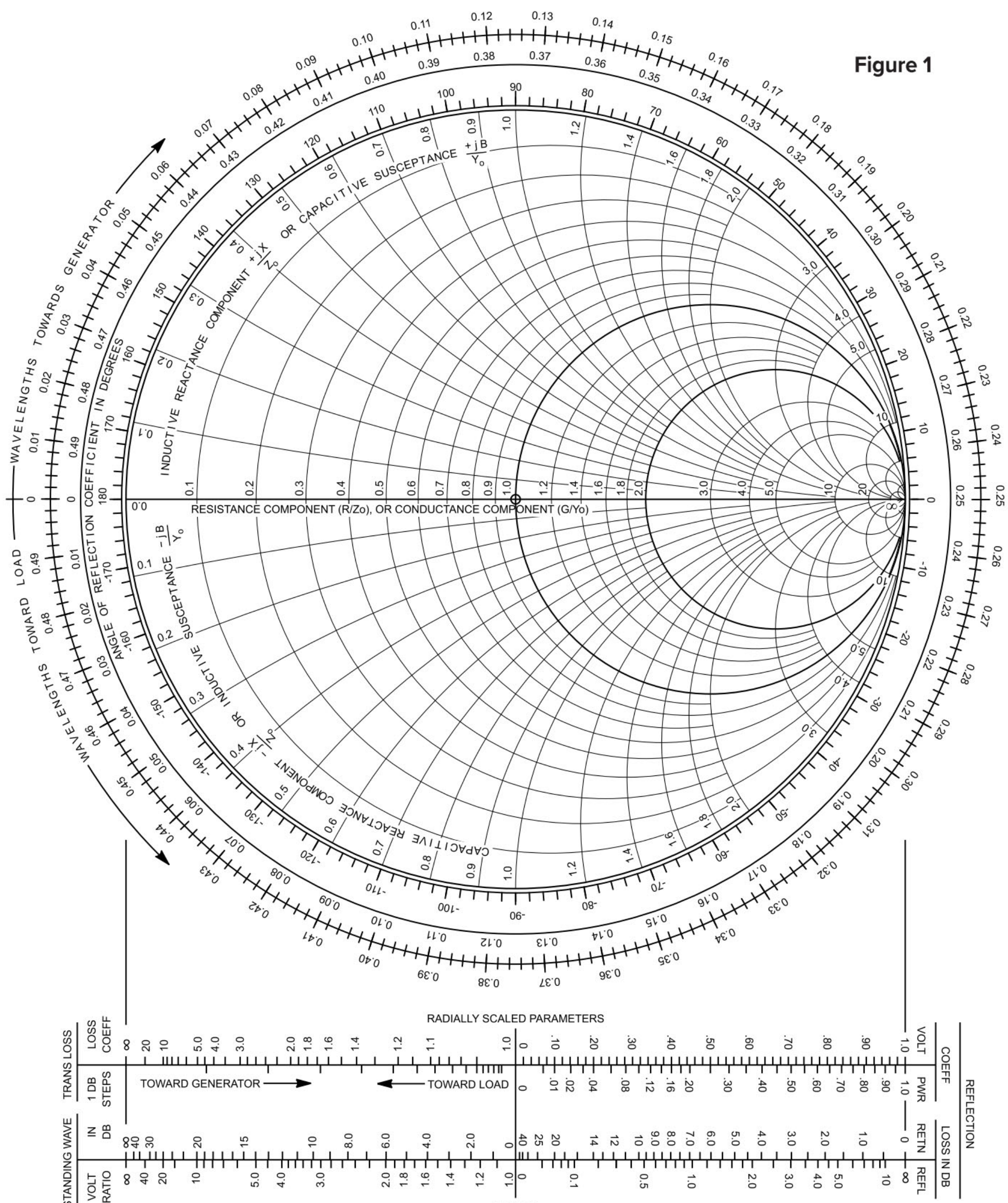
Self-Paced Essays – #19
The Smith Chart, Plain and Fancy

The Smith Chart is an easy way of performing transmission line calculations.

Figure 1



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In our previous *Essay*, we introduced the Vector Network Analyzer, an instrument so handy and inexpensive that there's really no excuse for any ham or electronics experimenter not to have one. My new NanoVNA just arrived yesterday; I had sold a slightly older version to a new ham who was very excited to play with this new technology. However, in keeping with my philosophy that it's always best to know the answer before you hand it over to a computer, we're first going to discuss the "hard way" of doing things with the Smith Chart, which is itself a major simplification of the "real hard way" of performing transmission line calculations.

To really appreciate the Smith Chart, let us first look at the equation (19) from which the Smith Chart was derived, see the 24th edition of the *ARRL Antenna Book*; here reproduced as **Eqn (1)**:

$$Z_{in} = Z_0 \frac{Z_L \cosh(\gamma l) + Z_0 \sinh(\gamma l)}{Z_L \sinh(\gamma l) + Z_0 \cosh(\gamma l)}$$

where:
Z_{in} = complex impedance at input of line
Z_L = complex impedance at end of line = $R_a \pm jX_a$
Z_0 = characteristic impedance of line = $R_0 - jX_0$
l = physical length of line
$\gamma = \alpha + j\beta$
α = matched loss attenuation constant in nepers/unit length-line (1 neper = 8.686 dB)
β = phase constant of line in radians/unit length, where 2π radians = one wavelength
and where:

$$\beta = \frac{2\pi}{VF \times 983.6 / f_{MHz}} \text{ for } l \text{ in feet}$$

VF = velocity factor.

Whenever my students complain about how hard the Smith Chart is, I simply refer them to the equations above and ask them if they want to do it this way. The Smith Chart always wins.

Now, in the previous *Essay*, we pointed out that the typical VNA is normalized to 50 Ω, since that is the characteristic impedance of the most common coaxial transmission line. The "full-fledged" Smith Chart, **Figure 1**, is non-normalized, which means you can use it with any impedance transmission line, with a couple of extra steps involved. The *ARRL Antenna Book* also has some high resolution charts you can work with in the chapter on transmission lines. Things can get a little "cramped" on the "adult version" of the Smith Chart.

Let's take a simple example of a quarter-wave length of 600 Ω ladder line, with a load consisting of a 50 Ω resistor and a 50 Ω inductor in series. Our task is to find out what the input impedance of the transmission line is. Using classic "*j* notation" our load impedance is (50 + *j* 50) Ω. The *j* is positive because it is inductive. First we need to normalize the impedance, which means we divide each term (the resistance and reactance) separately by the characteristic impedance of the line. In this case our normalized impedance is 50/600 +*j* 50/600, or 0.083 + *j* 0.083. Unless you use a really fine point on your pencil, you probably will not be able to resolve more than two significant digits. In fact, for this example, we'll use just (0.08 + *j* 0.08) Ω. That point is plotted near the left extreme of the chart of **Figure 2**.

Now, before we go any further, let's do one optional step that will really reveal a lot about this whole matter. Locate a compass (the circle making kind). Stick the pointy part of the compass

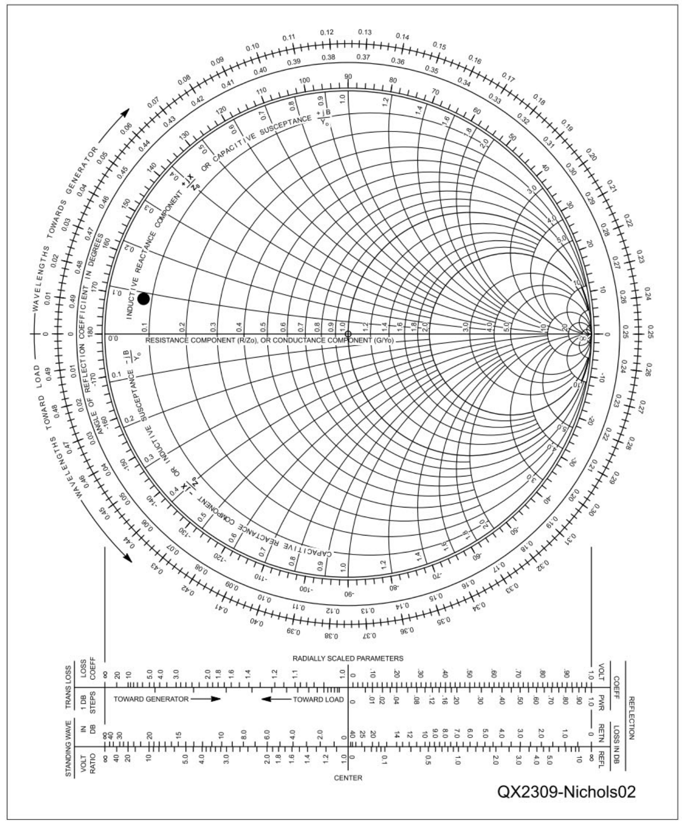

Figure 2

dead center of the chart (the origin), and place the pencil on the impedance point you plotted above. Now draw a complete circle around the chart, see **Figure 3**.

The circle you've just drawn is the SWR circle, which is the *locus* of every possible impedance that can exist on this particular transmission line. If you were to drop a line segment down from the very left extreme of the SWR circle (where it crosses the horizontal axis) on down to the "radial scaled parameters," you will be able to read the SWR directly, in this case, just a smidgen above 12:1 SWR. If there were no reactance in the load, the SWR would be *exactly* 12:1.

Now, let's take a straight edge, and draw a line segment from our original point, through the origin, and on through to where it intersects with the SWR circle on the "southeast" side (**Figure 4**).

The point of intersection is our new impedance, which is *about* 13 – *j* 5.0 ohms. Why the negative *j*? Because we are now below the "horizon," where all values are capacitive. But we still have one more step to perform. We need to *de-normalize* our impedance, which means we need to *multiply* each term by the characteristic impedance. This gives us 13×600 – *j* 5.0×600, or 7800 – *j* 3000 ohms, which is our input impedance.

This impedance isn't likely to make many transmitters happy. However, if we add *another* quarter wave of transmission line, this will bring our impedance right back to where we started, which is 50 + *j* 50 ohms, which isn't too bad a mismatch. For a typical 50 ohm transmitter, this will look like about a 2.5:1 SWR, not great, but usable.

You can, of course, use this method for any length of transmission line, using the wavelengths toward generator or wave-

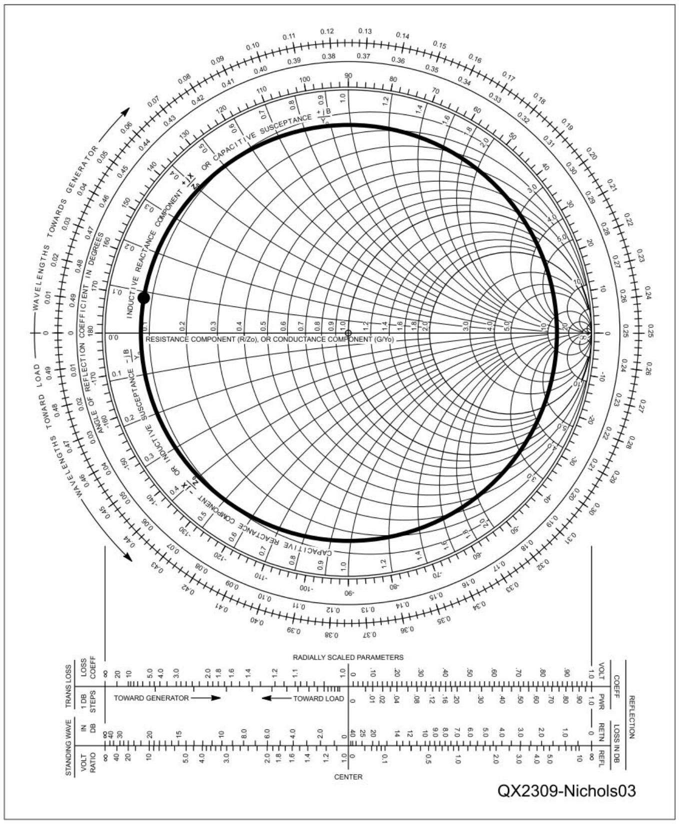

Figure 3

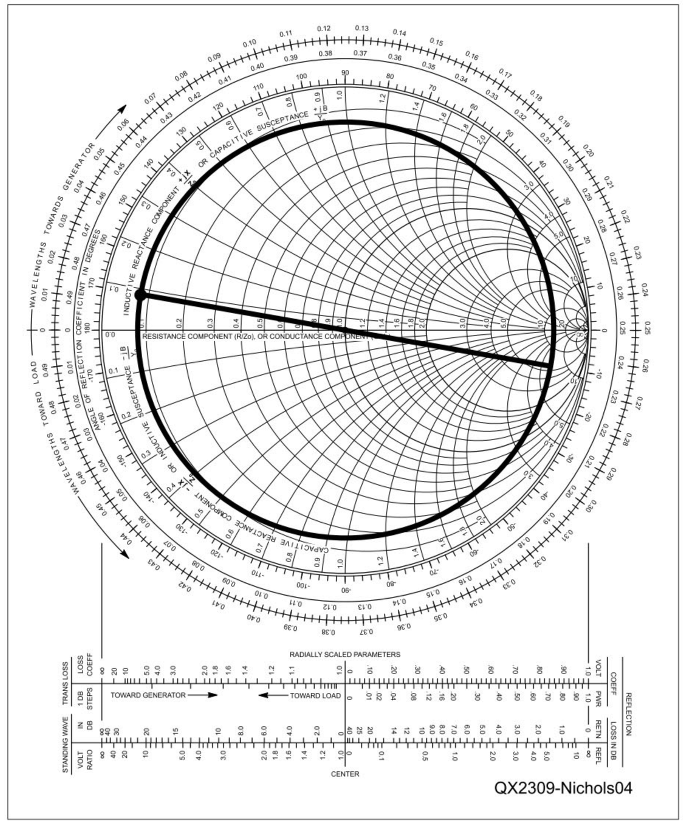

Figure 4

lengths toward load scales on the outer perimeter of the chart. Quarter wave increments are just simpler to follow, because you just go diametrically across the chart. Remember than any possible impedance will lie somewhere on your SWR circle. By the way, it is always best to convert frequencies to wavelengths first, before doing any Smith Chart calculations.

After a few transmission line calculations, the Smith Chart will become second nature. Really! Now with the ready availability of the NanoVNA, you can check your work very quickly.

Here are a few "self-evident truths" that the Smith Chart clearly presents, which can greatly increase your grasp of transmission lines.

1) The greater the SWR, the greater will be the radius of the SWR circle. From a practical standpoint, this means that the greater the SWR, the greater the *variation* of impedance as you move along the transmission line. A "flat" transmission line will simply be a point in the center of the Smith Chart.
2) Impedances change much more rapidly with respect to frequency near the right hand side of the Smith Chart. This graphically explains the "squirrely" behavior of high impedance antennas like the increasingly popular End Fed Half Wave (EFHW) antenna.
3) A purely reactive load will fall on the outer perimeter of the Smith Chart, meaning that with no resistance in the load, the SWR will always be infinite.

As always, we invite your comments and questions; we want to keep these *Essays* as interactive as possible. If we need to camp out a little longer on any particular topic, we can do that. Usually, the Smith Chart merits a few passes, as it is not only important, but fairly complicated to many folks. It is well worth understanding *completely.*

Next essay, we will talk a bit about antenna modeling, and the merits of verifying your antenna models with real world physical construction. — *73, until next time, Eric*.