ملاحظة: المكثف بهدف لتلخيص اهم النقاط للرياضيات المتقطعة, بس ما بظمنلك تجيب العلامة الكاملة, رح تستفيد من هاظ المكثف لو درست المادة قبل تدرس هان, و انا مش مسؤول عن اي حد ما جاب العلامة الي بدو اياها

Chapter 1: Logic

Propositional Logic (Calculus): It deals with propositions (statements that are either true or false (but not both)).

Propositions are denoted by letters, often P and Q.
True values are represented by T, and false values by F.

Propositions can be:

1. Atomic: Single proposition.
2. Compound: Multiple propositions linked by logical operators.

Truth Table

A truth table is a tool that lists all possible truth values of a logical statement, showcasing the effects of each logical operator and revealing the resulting truth value of the statement.

Logical Operators:

(will use q,p,r and as examples, but any letter works) **1.Negation:** Read as NOT P, and written as $\neg$ p, reverses the truth value of P

P	¬P
True	False
False	True

2.Conjunction: Read as Q and P, and written as Q∧P, is true if p and q are both true

P	Q	P ∧ Q
True	True	True
True	False	False
False	True	False
False	False	False

3.Disjunction: read as P OR Q, written as P ∨ Q, is only false if p and q are both false

P	Q	P ∨ Q
True	True	True
True	False	True

P	Q	P ∨ Q
False	True	True
False	False	False

4.Exclusive OR: read as P XOR Q, written as P ⊕ Q, is true if either P or Q are True, but not both

P	Q	P ⊕ Q
True	True	False
True	False	True
False	True	True
False	False	False

5.Implication: read as P implies Q, written as P→Q, is only false of P is true and Q is false,

P	Q	P → Q
True	True	True
True	False	False
False	True	True
False	False	True

6.Biconditional: read as P if and only if Q, written as P ⟺ Q. is only true if P and Q are the same value.

P	Q	P ↔ Q
True	True	True
True	False	False
False	True	False
False	False	True

Exercise: Solve (P ^ Q) v ~R

P	Q	R	(P ^ Q)	(P ^ Q) v ~R
T	T	T	T	T
T	T	F	T	T
T	F	T	F	F
T	F	F	F	T
F	T	T	F	F
F	T	F	F	T

P	Q	R	(P ^ Q)	(P ^ Q) v ~R
F	F	T	F	F
F	F	F	F	T

Components of Implication (P → Q):

1. **Converse:**
   - Original Statement: If it is raining, then it is cloudy. (P → Q)
   - Converse (Not the same): If it is cloudy, then it is raining. (Q → P)
2. **Inverse:**
   - Original Statement: If it is raining, then it is cloudy. (P → Q)
   - Inverse (Not the same): If it is not raining, then it is not cloudy. (¬P → ¬Q)
3. **Contrapositive:**
   - Original Statement: If it is raining, then it is cloudy. (P → Q)
   - Contrapositive (Same): If it is not cloudy, then it is not raining. (¬Q → ¬P)

Logical Operator Precedence:

1. **Negation (¬)** - Highest precedence.
2. **Conjunction (AND, ∧)**
3. **Disjunction (OR, ∨)**
4. **Conditional (→)**
5. **Biconditional (↔)** - Lowest precedence.

If the operators are the same, priority goes from left to right.

Exercise: Assume P: T Q: F R:F, Find the value of: P∨(Q∧¬R)⇔P
P∨(Q∧¬R)⇔P

T∨(F∧¬F)⇔T

T∨(F∧T)⇔T

T∨F⇔T

T ⇔T
T

Applications of Propositional Logic:

1. Translating logic expressions to English:

Examples:
I am hungry (p) if and only if (↔) I will eat (q) P↔Q

If (→) it is snowing (p), then I will wear a coat (q) $P \to Q$

The store is not closed $\neg P$

2. Bit-wise operations

Binary system: Utilizes bits, each with two values (0 or 1), representing true (1) or false (0).

Boolean variables: variables that can only hold true or false values.

Logical Operator	**Bit operator**
$\neg$	NOT
$\lor$	OR
$\land$	AND
$\oplus$	XOR

- Bit string: it is a sequence of zero or more bits.
- String Length: number of bits in the Bit string

e.i: 101010011 is a bit string with length = 9

Logical Equivalence

Tautology: compound proposition that is always true (Ex: P $\lor$$\neg$P)

p	**¬p**	**$P \lor \neg P$**
t	f	t
f	t	t

Contradiction: compound proposition that is always false (Ex: $P \land \neg P$)

p	**¬p**	**$P \land \neg P$**
t	f	f
f	t	f

Contingency: compound proposition that is either true or false (Ex: $P \to Q$)

Exercise: show that $\neg (P \lor Q) = (\neg P \land \neg Q)$ using truth table

P	**Q**	**$(P \lor Q)$**	**$\neg (P \lor Q)$**	**$(\neg P \land \neg Q)$**	**$\neg (P \lor Q) \iff \neg P \land \neg Q)$**
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	F	F	T
F	F	F	T	T	T

Equivalence rules:

Equivalence rule	Name
P ^ T = P P v F = P	Identity
P ^ F = F P v T = T	Domination
P ^ P = P P v P = P	Idempotent
¬ P v P = T ¬ P ^ P = F	Negation
~(~P) = P	Double Negation
P ^ Q = Q ^ P P v Q = Q v P	Commutative
(P v Q) v R = P v (Q v R) (P ^ Q) ^ R = P ^ (Q ^ R)	Associative
P v (Q ^ R) = (P v Q) ^ (P v R) P ^ (Q v R) = (P ^ Q) v (P ^ R)	Distributive
~(P v Q) = ~P ^ ~Q ~(P ^ Q) = ~P v ~Q	De Morgan's Law
P v (P ^ Q) = P P ^ (P v Q) = P	Absorption

Implications Logical Rules

note: there are others, these 2 are the most important ones

Statement 1	Statement 1
P →Q	~P v Q
P →Q	~Q →~P

Bicondintional Rules

note: there are others, this 1 is the most important ones

Statement 1	Statement 2
P ⟺ Q	(P →Q) ^ (Q→ P)

Example 1: Show that the following is a tautology.

¬ (P ∧ Q) → (P ∨ Q)

¬ (P ∧ Q) = (¬ P ∨ -Q)

(¬ P ∨ -Q) → (P ∨ Q) =(¬ P ∨ P) ∨ (¬ Q ∨ Q) (

T ∨ T = T

Example 2: Show that the following is logically equivalent.

¬(P∨(¬P ∧ Q) = (¬P ∧ ¬Q)

(¬P ∧ ¬(¬P ∧ Q)

(¬P ∧ (¬¬P ∨ ¬Q)

(¬P ∧ P) ∨ (¬P ∧ ¬ Q)

F ∨ (¬P ∧ ¬ Q)

(¬P ∧ ¬ Q)

PREDICATES AND QUANTIFIERS

predicates: statements that are not propositions

examples:

Ex2: Q(x, y): x= y+3

Ex3: R(X,Y,Z): X + Y = Z .

QUANTIFIERS:

1. Universal quantifier (∀), for all

* P(x) is true for all values of x in the universe of discourse (domain). ➔ ∀x p(x) * ∀x p(x) is read as: “for all x p(x) “ , “ for every x p(x)”

examples:

What is the truth value ∀x p(x), where p(x) is (x < 10). The domain is all positive integers not exceeding 4?

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Sol : ∀x p(x) = P(1) ^ p(2) ^ p(3) ^ p(4)
T ^ T ^ T ^ F = F
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2. Translate the following statement into English language:
   ∀x Q(x), where Q(x) is “x has two parents” and the domain is all people.
   Sol: every person has two parents

2. Existential Quantifier (∃), for some

* P(x) is true if an element (x) is true). ➔ ∃ p(x) * ∃ p(x)) is read as: “there is a x such that p(x) “,“ there is at least one x such that p(x)”

Example:

what is the truth value of ∃x p(x), where p(x) is “x * x > 10” and the domain is all positive integers
not exceeding 4?

∃x p(x) = P(1) ∨ p(2) ∨ p(3) ∨ p(4) = True, since p(4) is True

Binding Variable

A variable in a predicate might be:

1. Free: p(x): x has a cat
2. Bound to either
    1. to a value: p(Ali): Ali has a cat. (x is bound to ali)
    2. To a quantifier: ∀x ∃y like(x, y), x and y are bound to ∀ ∃ respectively

Negation

to negate we first change the Quantifier, then negate the inside statement example:

There is a student in the class who has taken Calculus. ∃x p(x)

becomes: Every student in the class has not taken calculus. ¬∃x P(x) = ∀ x ¬ P(x)

NESTED QUANTIFIER

tldr: more than one expression

example

∀x(∀y (x + y = y + x)) is true, for every values x and y x + y = y + x Domain: Real Numbers. Note: ∀x(∀y (x + y = y + x)) is the same as ∀x ∀y x + y = y + x (parentheses are optional)

to negate a nested quantifier we negate each quantifier then move to the other one, negating it as well

example: ¬∀x∀y ∃z (P(x,y) ∧Q(y,z)) → ∃x¬∀y ∃z (P(x,y) ∧Q(y,z)) → ∃x ∃y ¬∃z (P(x,y) ∧ Q(y,z)) → ∃x ∃y ∀z¬ (P(x,y) ∧Q(y,z)) → ∃x ∃y ∀z(¬P(x,y) ∨¬Q(y,z))

Chapter 2

Sets

An unordered collection of objects.

The objects in a set are called the elements, or members, of the set. A set is said to contain its elements

$S = \{a, b, c, d\}$

We write ($a \in S$) to denote that a is an element of the set S. The notation $e \notin S$ denotes that e is not an element of the set S.

Another way to describe a set is to use set builder notation

The set O of odd positive integers less than 10 can be expressed by $O = \{1, 3, 5, 7, 9\}$.

OR

$O = \{x \mid x \text{ is an odd positive integer} < 10\}$ / $O = \{x \in Z^{+} \mid x \text{ is odd and } x<10\}$

List of Unique sets

Letter	Represents
N	Natural Numbers (0-infinity)
Z	All integers
Z^{+}	All Positive integers
Q	All Rational Numbers
R	Real Numbers
R^{+}	Positive Real Numbrers
C	Complex Numbers

Interval Notation

1. Closed interval $[a, b]$
2. Open interval (a, b)

Interval	Implication
$[a, b]$	$\{x \mid a \leq x \leq \mid b\}$
$[a, b)$	$\{x \mid a \leq x < \mid b\}$
$(a, b]$	$\{x \mid a < x \leq \mid b\}$
(a, b)	$\{x \mid a < x < b\}$

If A and B are sets, then A and B are equal if and only if

$\forall x\ (x \in A \leftrightarrow\ x \in B)$. We write $A = B$, if A and B are equal sets

The sets {1, 3 , 5} and {3, 5 , 1} are equal, because they have the same elements.

{1 , 3 , 3 , 5 , 5 , 5} is the same as the set {1, 3 , 5} because they have the same elements

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Empty Set/ Null set

A set that has no elements, is denoted by ∅ or by { }.

Cardinality

The cardinality is the number of distinct elements in S. The cardinality of S is denoted by $|S|$.

examples:
A = {1, 2, 3, 7, 9}
$|A|$ = 5

∅ = {}
|∅| = 0

A = {1, 2, 3, {2,3}, 9}

$|A|$ = 5 ({2,3} is counted as one)

{∅} = {{}}

|{∅} | = 1

Infinite

A set is said to be infinite if it is not finite. The set of positive integers is infinite.

example: Z = {0,1,2,3............}

Subset

The set A is said to be a subset of B if and only if every element of A is also an element of B .

We use the notation $A \subseteq B$ to indicate that A is a subset of the set B .

$A \subseteq B\ \leftrightarrow\ \forall x(x \in A \rightarrow x \in B)$

$(\boldsymbol{A} \subseteq \boldsymbol{B}) \equiv (\boldsymbol{B} \supseteq \boldsymbol{A})$

For the set S

1. ∅ ⊆ S
2. S ⊆ S

To show that two sets A and B are equal, show that $A \subseteq B$ and $B \subseteq A$.

Proper Subset

The set A is a subset of the set B but that $A \neq B$, we write $A \subset B$

and say that A is a $\mathbf{proper\ subset}$ of B.

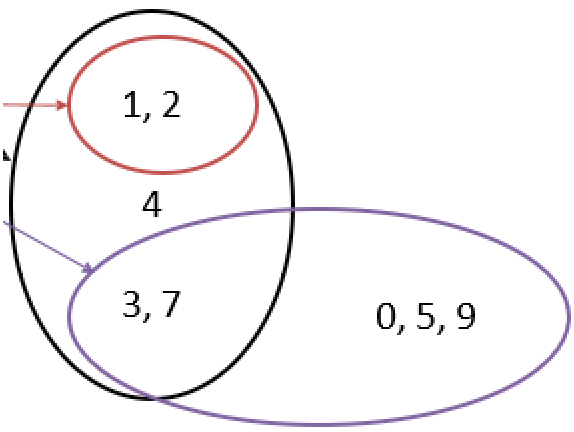
$A \subset B \leftrightarrow (\forall x \;\; x \in A \rightarrow x \in B) \land \exists x \, (x \in B \land x \notin A)$

Venn Diagram

$A =$ 1,2,3,4,7 (black)

$B =$ 0,3,5,7,9 (purple)

$C =$ 1,2 (red)



Power Set

$\mathbf{The\ set\ of\ all\ subsets.}$

If the set is S. The power set of S is denoted by $P(S)$. The number of elements in the power set is 2^S

example:
S = {1,2,3} = {∅, 1 , 2 , 3 , 1,2 , 1,3 , 2,3 , 1,2,3}

$P(S) = 2^S = 2^3 = 8$

The power set of an empty set is

p(∅) = {∅}

The power set of the set {∅} is

P({∅}) = {∅, {∅}}

The ordered n-tuple

The ordered collection that has $a1$ as its first element, $a2$ as its second element, … , and an as its nth element. ($a1$, $a2$,.....an)

ordered 2-tuples are called ordered pairs (e.g., the ordered pairs (a, b))

Cartesian Products

the set of all ordered pairs (a, b), where $a \in A$ and $b \in B$ denoted by $A \times B$

Example:

A = {1,2}, B = $\{a, b, c\}$

$A \times B$ = $(1, a)$, $(1, b)$, $(1, c)$, $(2, a)$, $(2, b)$, $(2, c)$.

$|A \times B|$ = $|A| * |B|$ = |2 ∗ 3| = |6|

The Cartesian product of more than two sets.

A X B x C, where A = {0, 1 } B = {1, 2}, and C = {0, 1, 2

A x B × C ={(0,1,0),(0,1,1),(0,1,2),(0,2,0),(0,2,1),(0,2,2),

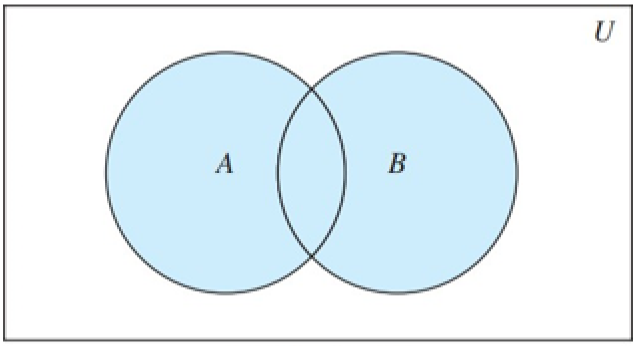
(1,1,0),(1,1,1),(1,1,2),(1,2,0),(1,2,1),(1,2,2)}.

Set Operations

Unions

The set that contains elements that are either in A or in B , or in both.

$A \cup B = \{x \mid x \in A \vee x \in B\}$



$A \cup B$ is shaded.

Example:

The union of {1, 3, 5} and {1, 2, 3} is {1, 2, 3, 5}

Unions Can Be Generalized

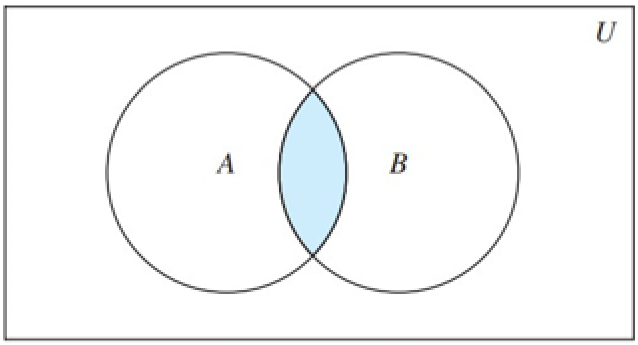
using the notation:

$$A_1 \cup A_2 \cup \cdots \cup A_n = \bigcup_{i=1}^{n} A_i$$

Intersection

The set that contains those elements that are in both A and B.

$A \cap B = \{x \mid x \in A \wedge x \in B\}$



$A \cap B$ is shaded.

The intersection of {1, 3, 5} and {1, 2, 3} is set {1, 3}

Two sets are called disjoint if their intersection is the empty set.

$A \cap B = \emptyset$

Intersections Can Be Generalized

using the notation:

$$A_1 \cap A_2 \cap \cdots \cap A_n = \bigcap_{i=1}^{n} A_i$$

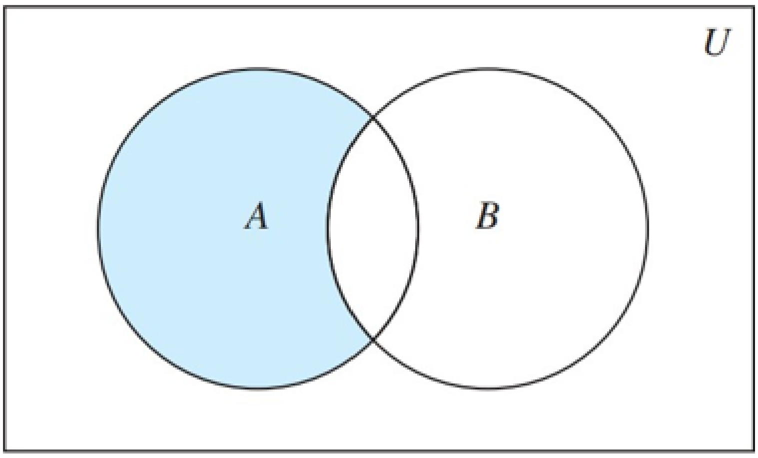
Difference

The set containing elements that are in A but not in B

A - B={x |x ∈ A ∧ x ∉ B}

A = {1,3,5} B= {1,2,3}

A - B = {5}



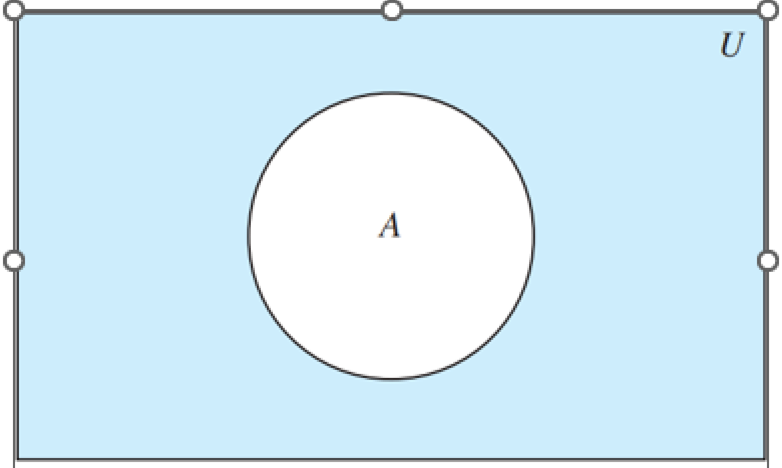
$A - B$ is shaded.

Complement

An element x belongs to U (a universal set) if and only if $x \notin A$

U = {1,2,3,4,5} , A = {1,3}

Aпъ = {2,4,5}



$\bar{A}$ is shaded.

Set Identities

TABLE Set Identities.

Identity	*Name*
$A \cap U = A$ $A \cup \emptyset = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\bar{A})} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws

TABLE Set Identities.	
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement laws

(same as the ones in logic)

Exercise:

Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

Can be solved 2 ways

$$\overline{A \cap B} \subseteq \overline{A} \cup \overline{B}.$$

$x \in \overline{A \cap B}$	by assumption
$x \notin A \cap B$	defn. of complement
$\neg((x \in A) \wedge (x \in B))$	defn. of intersection
$\neg(x \in A) \vee \neg(x \in B)$	1st De Morgan Law for Prop Logic
$x \notin A \vee x \notin B$	defn. of negation
$x \in \overline{A} \vee x \in \overline{B}$	defn. of complement
$x \in \overline{A} \cup \overline{B}$	defn. of union

OR

$$\overline{A} \cup \overline{B} \subseteq \overline{A \cap B}.$$

$$
\begin{array}{ll}
x \in \overline{A} \cup \overline{B} & \text{by assumption} \\
(x \in \overline{A}) \vee (x \in \overline{B}) & \text{defn. of union} \\
(x \notin A) \vee (x \notin B) & \text{defn. of complement} \\
\neg(x \in A) \vee \neg(x \in B) & \text{defn. of negation} \\
\neg((x \in A) \wedge (x \in B)) & \text{by 1st De Morgan Law for Prop Logic} \\
\neg(x \in A \cap B) & \text{defn. of intersection} \\
x \in \overline{A \cap B} & \text{defn. of complement}
\end{array}
$$

We can also solve them using set builder notations.

Function

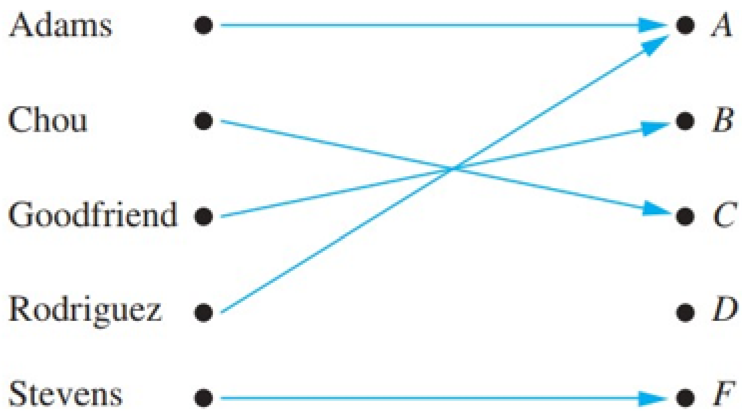
A and B = nonempty sets

function f from A to B is an assignment of exactly one element of B to each element of A.

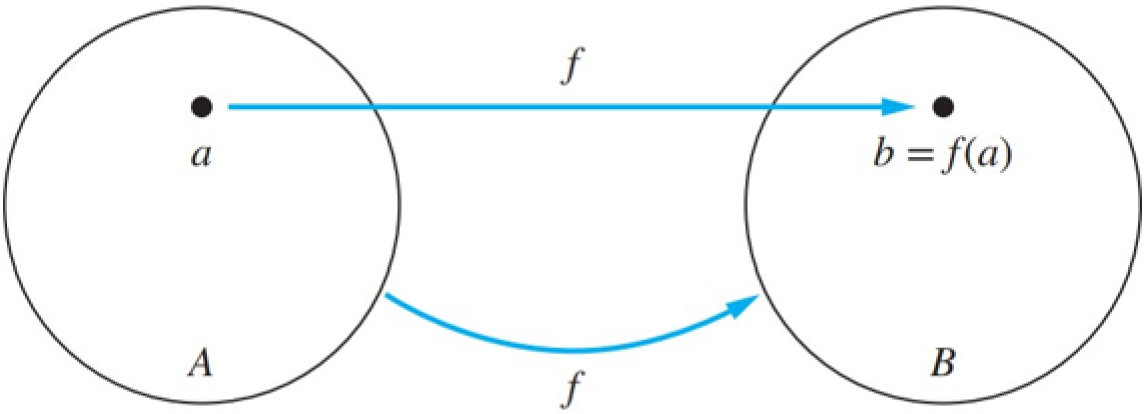
We write $f(a) = b$ if b is the unique element of B assigned by the function f to the element a of A.

If f is a function from A to B, we write $f: A \rightarrow B$

example:



Assignment of grades in a discrete mathematics class.



The function f maps A to B.

Domain: A

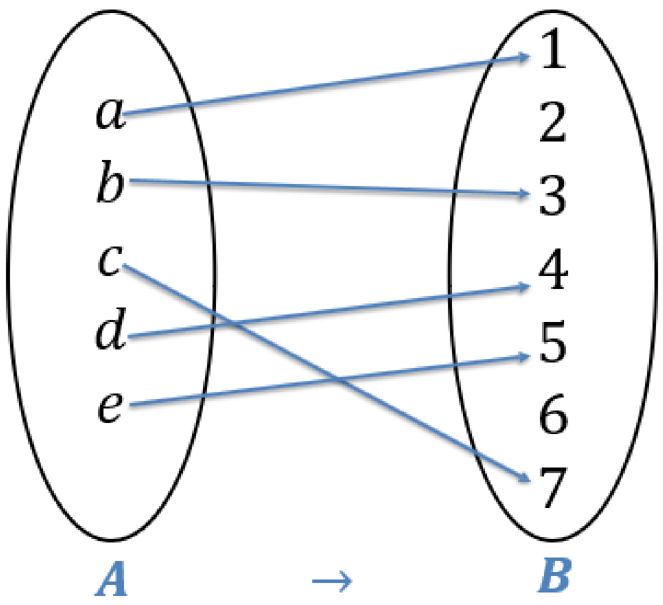
Co-Domain: B

$f\ a\ = b$

b is the image of a

a is a preimage of b

The range, or image, of f is the set of all images of elements of A.



Domain = $\{a, b, c, d, e\}$

Co-Domain = $\{1,2,3,4,5,6,7\}$

Range = $\{1,3,4,5,7\}$

Let $f1$ and $f2$ be functions from A to R. Then $f1 + f2$ and $f1\ f2$ are also functions from A to R defined for all $x \in A$ by

$f1 + f2\ \ (x) = f1(x) + f2(x)$, $(f1f2)(x) = f1\ \ x\ \ f2(x)$.

example:

$f1(x) = x^2$ and $f2(x) = x - x^2$.

What are the functions $f1 + f2$ and $f1f2$?

(f1+f2)(x)=f1(x)+f2(x)=x^2+(x-x^2)=x

(f1f2)(x) =f1(x)f2(x) =x^2(x-x^2)=x^3-x^4.

---

f = function from A to B.

S = subset of A.

The image of S under the function f is the subset of B that consists of the images of the elements of S.

Denoted by:

$f(S) = \{ t \mid \exists s \in S\ \ (t = f\ \ (s))\ \}$.

or shortly $\{f\ \ s\ \mid s \in S\}$.

example:

A = {a, b, c, d, e} B = {1, 2, 3, 4} $f(a)$ = 2, $f(b)$ = 1, $f(c)$ = 4, $f(d)$ = 1, $f(e)$ = 1.

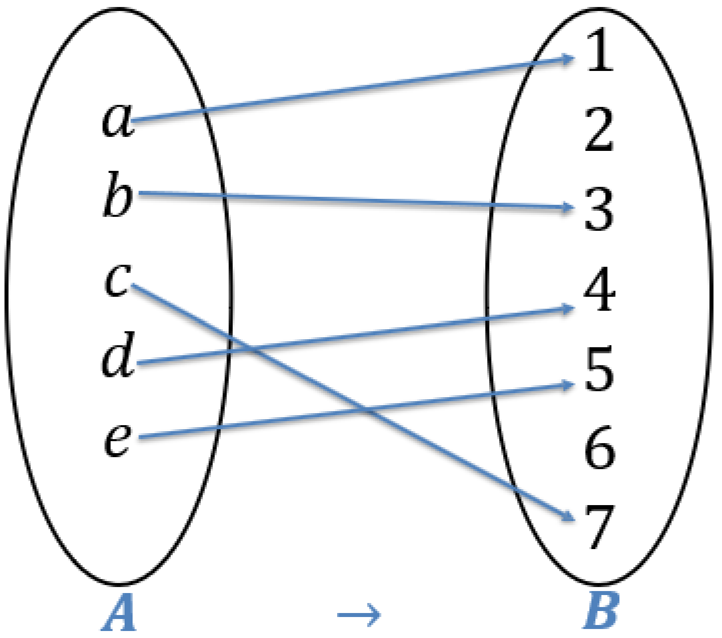
S = {b, c, d} $\subseteq A$

image of S = {b, c, d} is $f(S)$ = {1, 4}

One-to-One function (injective)

if f(a) = f(b) implies that a = b for all a and b in the domain of f.

(every case is unique / no 2 f(a) are the same)
example:



$f(a) = 1$

$f(b) = 3$

$f(c) = 7$

$f(d) = 4$

$f(e) = 5$

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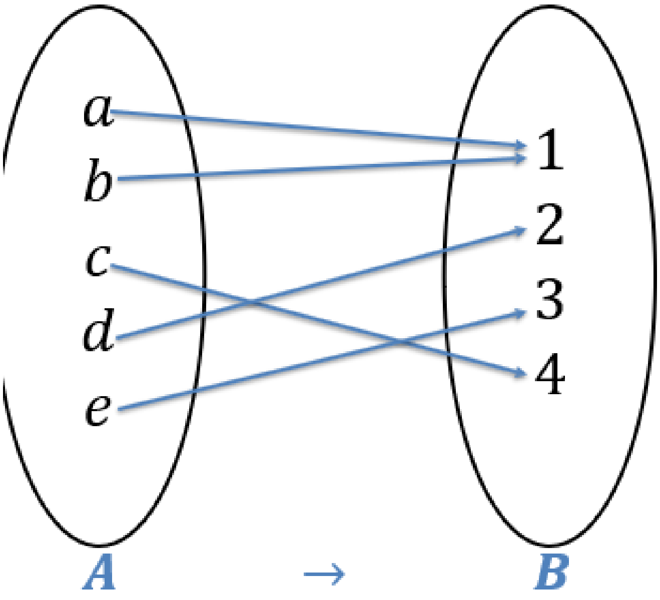
onto function (surjective)

If and only if for every element b ∈ B there is an element a ∈ A with f(a) = b.

(every f(a) links to f(b))

example:



$f(a) = 1$

$f(b) = 1$

$f(c) = 4$

$f(d) = 2$

$f(e) = 3$

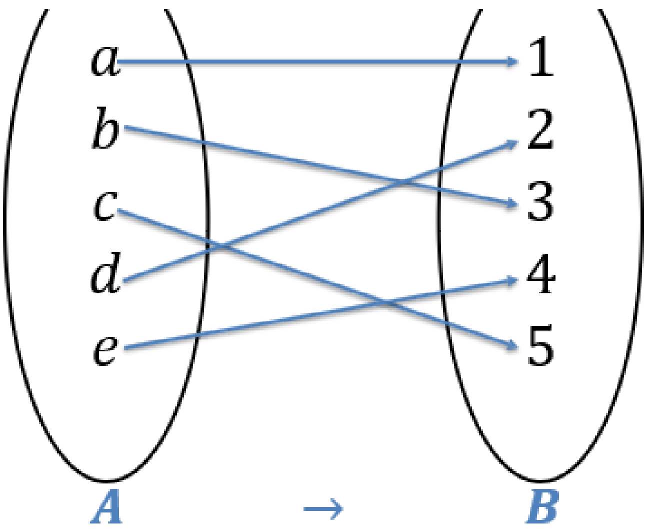
Co-Domain = {1,2,3,4}

Range = {1,2,3,4}

One-to-one correspondence (bijection)

if it is both one-to-one and onto.

example:



$f(a) = 1$

$f(b) = 3$

$f(c) = 5$

$f(d) = 2$

$f(e) = 4$

Co-Domain = {1,2,3,4,5}

Range = {1,2,3,4,5}

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exercise:

Determine if $f(x) = x + 1$ from set of ints to set of ints is 1 to 1

$f\ (a)\ = (a + 1)\quad f\ (b)\ = (b) + 1$

$a + 1 = b + 1$

a = b + 1 - 1

a = b

$\therefore f\ (x)$ is one−to−one

Determine if $f(x) = x^2$ from set of ints to set of ints is 1 to 1

$f\ (a)\ = a^2 \quad f\ (b)\ = b^2$

$a^2 = b^2$

$\pm a = \pm b$

a may be not equal b

$\therefore f\ \ x$ is NOT one−to−one

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determine if f(x) = (2x-1)/3 is onto

f(x) = y

(2x-1)/3 = y

2x-1 = 3y

2x = 3y+1

x = (3y+1)/2

f(x) =y

(2x -1)/3 = y

2 ((3y+1-1)/2)/3 = y

(3y+1-1)/3 = y

3y/3 = y

y = y

$\therefore f\ \ x$ is onto

Inverse Functions

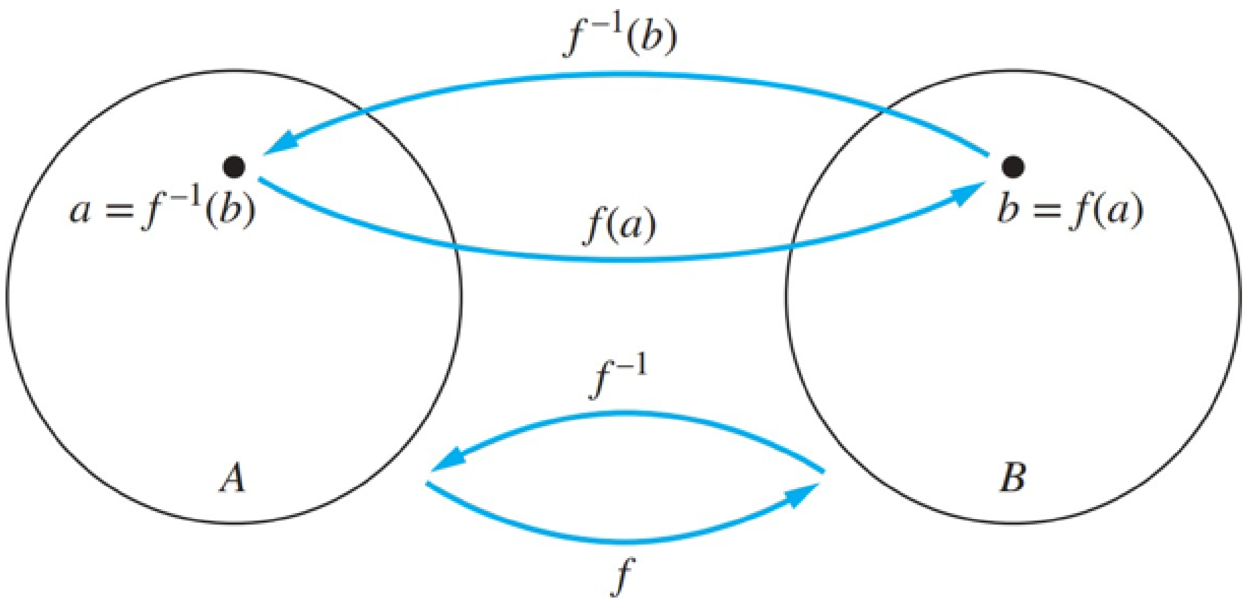
f = one-to-one correspondence from the set A to the set B.

The inverse function of f= function that assigns to an element b belonging to B the unique element a in A.

basically

$f(a) = b$

$f^{-1}(b) = a$



A one-to-one correspondence is called invertible because we can define an inverse of this function.

A function is not invertible if it is not a one-to-one correspondence, because the inverse of such a

function does not exist.

Example:

f =function from $\{a, b, c\}$ to { 1 , 2, 3}

$f(a) = 2$, $f(b) = 3$, $f(c) = 1$

Is f invertible, and if it is, what is its inverse?

f is invertible because it is a one-to-one correspondence.

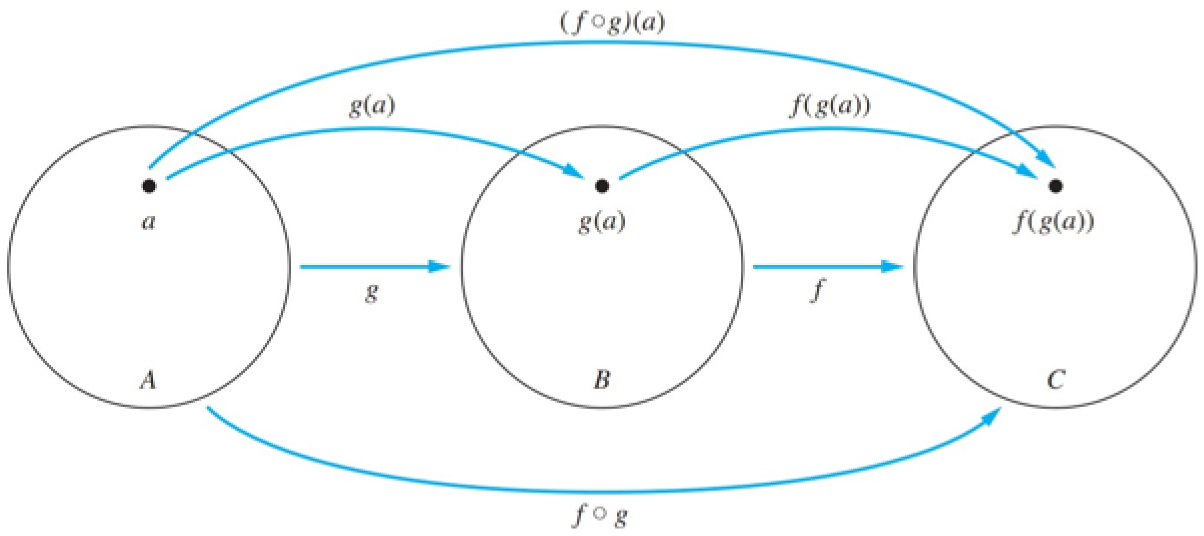
$f^{-1}(1) = c$, $f^{-1}(2) = a$, and $f^{-1}(3) = b$

Composition of the Functions f and g

g = function from the set A to the set B

f = function from the set B to the set C

The composition of the functions f and g, denoted by $f \circ$ g, is defined by ($f \circ$ g) a = (f g (a)).



Note: the composition f ∘ g cannot be defined unless the range of g is a subset of the domain of f .

Example:

g = function from the set {a , b, c} to itself

g(a) = b, g(b) = c, and g(c) = a.

f = function from the set {a , b , c} to the set { 1 , 2 , 3 }

f(a) = 3 , f(b) = 2 , and f(c) = 1

What is the composition of f and g, and what is the composition of g and f ?

1)The composition of f and g (i.e., (f ∘ g)): (f ∘g)(a) = 2, (f ∘g)(b) = 1, (f ∘g)(c) = 3

2)The composition of g and f (i.e., (g ∘ f)) cannot be defined because the range of f is NOT a subset of the domain of g .

another example

f and g: functions from the set of integers to the set of integers f(x) = 2x + 3 g(x) = 3x + 2

find the composition of f and g and the composition of g and f ?

1)The composition of f and g (i.e., (f ∘g))

(f ∘g)(x) = $f(g(x)) = 2(3x + 2) + 3 = 6x + 7$

2)The composition of g and f (i.e., (g ∘f))

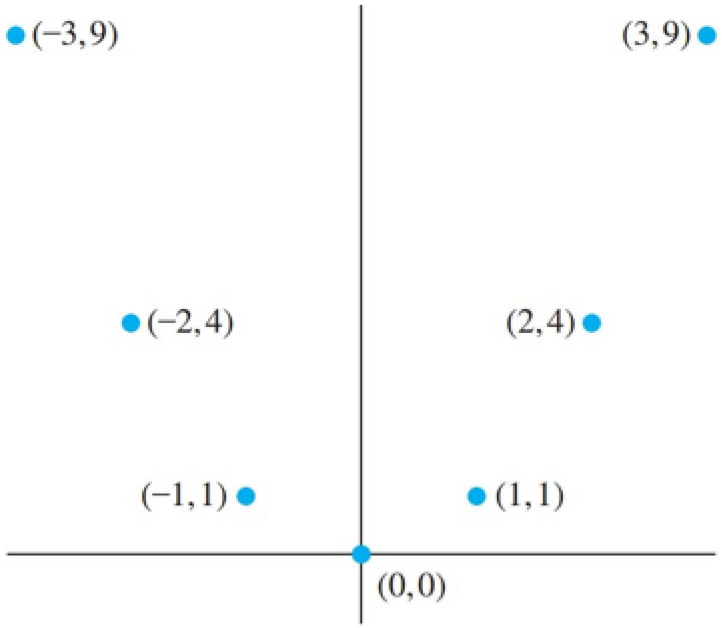
(g ∘f)(x) = $g(f(x)) = 3(2x + 3) + 2 = 6x + 11$

Graph of functions

f = function from A to B.

The graph of the function f is the set of ordered pairs $\{(a, b) | a \in A$ and $b \in B\}$.

example:



The graph of $f(x) = x^2$ from Z to Z.

Some Important Functions

1. Floor function (y = $\lfloor x \rfloor$)
   take a real number, and give the biggest integer that's smaller than that number
   examples:
   $\lfloor 2.5 \rfloor = 2$
   $\lfloor -2.5 \rfloor = -3$
2. Ceiling function (y = $\lceil x \rceil$)
   take a real number, and give the smallest integer that's bigger than that number
   examples:

⌈2.5⌉ = 3
⌈-2.5⌉ = -2

Useful Properties

1. ⌊-x⌋ = - ⌈_x_⌉ 2. ⌈- _x_⌉ = - ⌊x⌋ 3. ⌊x + n⌋ = ⌊x⌋ + n 4. ⌈_x_ +n⌉ = ⌈_x_⌉ +n

examples:

1. ⌊.5⌋ = 0
2. ⌈-1.2⌉ = -1
3. ⌊0.3 + 2 ⌋ = ⌊0.3 ⌋ + 2 = 0+2 =2
4. ⌈1.1+ ⌈0.5⌉⌉ = ⌈1.1⌉+ ⌈0.5⌉ = 2 + 1 = 3

Chapter 3

Relations

- Relation: relationships between elements of sets
- Relations are is just a subset of the Cartesian product of the sets.
- Binary relations: sets of ordered pairs
- The most direct way to express a relationship between elements of two sets is to use ordered pairs made up of two related elements.

A and B = sets

binary relation from A to B is a subset of $A \times B$. = a set R of ordered pairs, where first element is a and the 2nd element is b

We use $a\ R\ b$ to denote that $(a, b) \in R$, and $a \not R b$ to denote that $(a, b) \notin R$.

we also say a is said to be related to b by R when (a, b) belongs to R

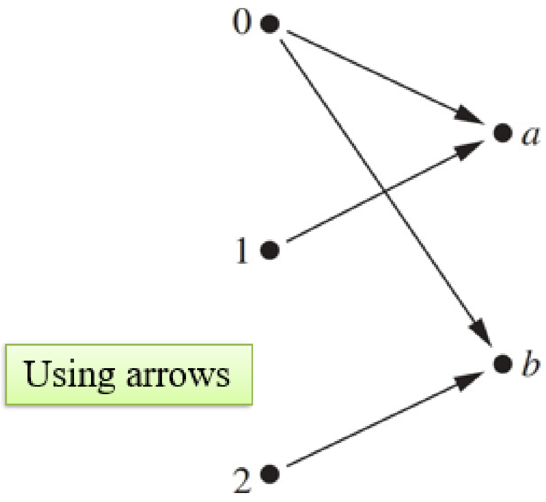
example

A = {0, 1 , 2}

B = $\{a , b\}$.

Roster notation = Roster form of set (denoted by R)
R = {$(\mathbf{0}, \boldsymbol{a})$, $(\mathbf{0}, \boldsymbol{b})$, $(\mathbf{1}, \boldsymbol{a})$, $(\mathbf{2}, \boldsymbol{b})$} = a relation from A to B

we can also denote them using the following



R	a	b
0	×	×
1	×	
2		×

Using table

Functions as Relations

function f from a set A to a set B assigns exactly one element of B to each element of A.

The graph of f is the set of ordered pairs (a , b) such that $b = f(a)$. (explain why)

Because the graph of f is a subset of $A \times B$, is a relation from A to B.

Relations on a Set

relation on the set A: a relation from A to A

or a relation on a set A: a subset of $A \times A$.

The identity relation I_A on a set A is the set $\{a, a \ \{a \in A\}\}$

(we take element of a and b, that fit the criteria)

Example = A = {1, 2, 3}

I_A = {(1, 1) , (2, 2) , (3, 3)}

example:

A = set {1, 2, 3 , 4}.

Which ordered pairs are in the relation $R = \{(a , b) | a$ divides $b\}$ --> (note: b/a not the other way, also it could be $a = b$ or $a > b$ or $a < b$)

solution = we need to find all the pairs where b/a is an int

{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (3,4)}

another example

A = {−1, 0, 1, 2}

Which ordered pairs are in the relations

R_1 = $\{(a , b) | a < b\}$

= { (−1, 0) , (−1, 1) , (−1, 2) , (0, 1) , (0, 2) , (1, 2) }

R_2 - {{a,b}|a > b }

= {(0,-1), (1,0), (1,-1),(2,1), (2,0),(2,-1)}

R_3 - {{a,b}| a = b}

= {(-1,-1), (0,0), (1,1), (2,2)}

R_4 - {{a,b}| a = $-b$}

= {(-1,1),(0,0),(1,-1)}

R_5 - {{a,b}| a = b or a = $-b$}

= (−1, −1) , (0, 0), (1, 1) , (2, 2) , (−1, 1) , (1, −1)

R_6 - {{a,b}| 0 ≤ a + b ≤ 1 }

= { (−1, 1) , (−1, 2) , (0, 0) , (0, 1) , (1, −1) , (1, 0) , (2, −1)}

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number of relations on set with n elements

because a relation on a set A is simply a subset of $A \times A$.

we can determine the number of subsets on a finite set using the following

$A \times A$ = A^2 = n^2

to determine the number of relations on set we use the following formula

2^{n^2}